**Problem 1.** *For $n$ sets $X_1, X_2, \ldots, X_n$ we define the $n$-*fold *product $X_1 \times X_2 \times \cdots \times X_n$ as the set*

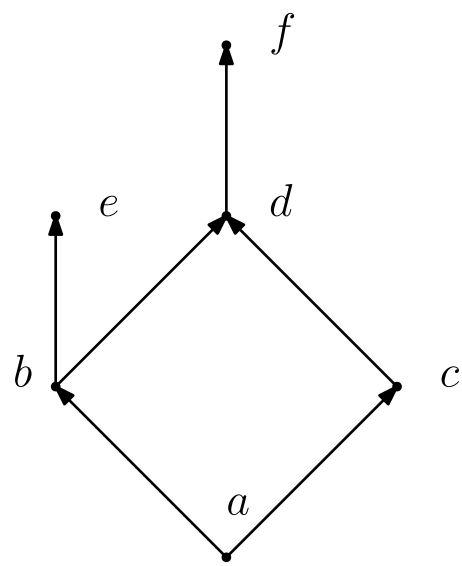
$$\prod_{i=1}^{n} X_i = \{(x_1, x_2, \ldots, x_n) : x_i \in X_i \ \forall i \in \{1, 2, \ldots, n\}\}$$

*Let $A = \{1, 2\}, B = \{a, b\}, C = \{\mu, \varepsilon\}$, write down the set $A \times B \times C$.*

**Problem 2.** *When is the following statement true?:*
*Let $X, Y, Z$ be sets. If $X \times Y = Z \times Y$ then $X = Z$.*
*Prove correctness of your answer.*

**Problem 3.** *Consider the relations on people "is a brother of", "is a sibling of", "is a parent of", "is married to", "is a descendant of". Which of the properties of reflexivity, symmetry, antisymmetry and transitivity do each of these relations have?*

**Problem 4.** *The picture below describes a relation $R$, where $(x, y) \in R$ means that there is a way to get from $x$ to $y$ using the arrows. Write down the relation $R$ (as a set). Which of the properties of reflexivity, symmetry, anti-symmetry, and transitivity does the relation have?*




**Problem 5.** *Determine which of the following relations are reflexive, symmetric, transitive or antisymmetric.*

1. $X = \{a, b, c\}, R = \{(a, a), (a, b), (b, b), (b, a), (c, c)\}$
2. $X = \{a, b, c\}, R = \{(a, a), (c, c)\}$
3. $(X, R) = (\mathbb{N}, \leqslant)$,

4. $X = \{1, 2, \ldots, 10\}, R = \{(x, y) : gcd(x, y) = 1\}$

5. $X$ *arbitrary,* $R = \emptyset$

6. $X$ *arbitrary,* $R = X \times X$

**Problem 6.** *Let $X$ be a set. Characterize all relations over $X$ that are both an equivalence and an ordering.*

**Problem 7.** *Let $R, S$ be two relations on a set $X$. Which of the following inherit the properties of reflexivity, symmetry, antisymmetry, transitivity? Prove your claims!*

1. $R \cup S$

2. $R \cap S$

**Composition of relations** Let $R$ be a relation from $X$ to $Y$ and let $S$ be a relation from $Y$ to $Z$. The composition $R \circ S$ is defined to be the relation

$$\{(x, z) \mid \exists y \in Y : (x, y) \in R, (y, z) \in S\}.$$

**Problem 8.** *Let $A = \{1, 2, 3\}, R = \{(1, 2), (2, 3)\}, \quad S = \{(2, 1), (3, 2)\}$.*

1. *Compute $S \circ R$ and $R \circ S$.*

2. *Decide which of $S \circ R$, $R \circ S$ are reflexive, symmetric, transitive on $A$.*

**Problem 9.** *Let $R$ be a relation on a set $X$. Prove that $R$ is transitive if and only if $R \circ R \subseteq R$*

**Problem 10.** *Let $R, S$ be two equivalence relations on a set $X$. Prove that $R \circ S$ is an equivalence relation if and only if $R \circ S = S \circ R$*