Problem set for Probability and Statistics — 9 March 2026

Random variables

1. Three friends decide to go swimming on some day of a given week but do not agree on a day in advance. So, each one shows up at the swimming pool on a (uniformly) random day, independently. Consider the random variable X to be the number of friends (from these three) who went on Friday. Find the probability distribution of X. Generalize this to n friends.

2. Shaq shoots a basketball at a basket, on each shot he has a probability 1/10 of scoring, the shots are independent. He quits after the first time he scores. Let X denote the total number of shots.

(a) What is $\mathbb{P}[X > k]$? (Try first for $k = 1$, $k = 2$.)

(b) What is the distribution of X? That is, determine the probability function p_X, i.e. for each x determine $\mathbb{P}[X = x]$ (do you know the name of this distribution?).

(c) What is $\mathbb{P}[X \geq 10 | X \geq 5]$?

3. Continuing from the last problem: let us denote $Y = X \bmod 2$, i.e. $Y = 0$ if X is even, otherwise $Y = 1$. Determine the distribution of Y.

4. Beatrice also shoots a basketball, she has a probability p of hitting the basket. Let Z denote the number of hits from n attempts. Determine the distribution of Z.

5. Let X and Y be discrete random variables on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and let $f : \mathbb{R} \to \mathbb{R}$ be a function. Prove that $f(X)$ and $X + Y$ are discrete random variables.

6. Find the cumulative distribution functions for the following distributions:

(a) $X \sim \mathsf{Bern}(p)$, i.e. the distribution with $p_X(0) = 1 - p, p_X(1) = p$;

(b) $X \sim \mathsf{Unif}(n)$, i.e. the distribution with $p_X(i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$;

(c) $X \sim \mathsf{Geom}(p)$, i.e. the distribution with $p_X(i) = p(1-p)^{i-1}$ for all positive integers i.

Bonus problems

7. (St. Petersburg Casino) We flip a coin repeatedly. If the first time the coin comes up heads on the nth flip, we get a reward of 2^n Kč. How much would you be willing to pay to participate in this game?

More practice problems

8. There are 100 coins in the chest. 99 of them are normal, but one has an eagle (which counts as tails) on both sides. We pull out a random coin and flip it six times, each time it lands tails. What is the probability that we have pulled a two-eagle coin? (Try to guess first, then calculate.)

9. In an election, people vote for two candidates, A and B. When leaving the polling station voters are randomly asked to participate in an exit poll. Assume that whoever answers will answer truthfully who they voted for, but not everyone will participate. If we denote by E the set of voters who participate in the exit poll, then suppose $\mathbb{P}[E|A] = 0.7$ and $\mathbb{P}[E|A^c] = 0.4$. The exit-poll results are 60 % for A. What is the actual proportion of people who voted for A?

10. We use smoke signals to transmit a binary file. Therefore, there is a relatively high probability of error for each bit: 0 is transmitted as 0 only with probability 0.9, 1 is transmitted as 1 only with probability 0.8. Assume (somewhat unreasonably) that each character is transmitted independently. Assume further that the 0s and 1s occur in each position independently and with equal probability.

(a) If we received a 0 signal, what is the probability that it was actually sent?

(b) We received message 0010. What is the probability that it was actually sent?

(c) How does the calculation change if we send each symbol three times to check (and then take the more frequent of the three attempts)?

11. The antlion is a parasite that attacks ants. It forces the attacked ant to climb to the top of a blade of grass and wait there to be eaten by a cow, in whose body it continues its development. All ants in the meadow are either infected or uninfected (i.e. healthy), and are either on the top of a blade of grass or on the ground. Of all the infected ants, 80% are on the top of a blade of grass. Of the healthy ants, only 10% are on the top of a blade of grass. A naturalist decided to find out what percentage of ants are infected. He did not want to bend down to the ground and dig in the mud, so he only collected ants that were on the top of the grass blades. Of these, 60% were infected with the antlion. What percentage of all ants are infected with the antlion?