Problem set for Probability and Statistics 1 — 2 March 2026

Conditional probability, Bayes' theorem

1. Does $\mathbb{P}[A|B] > \mathbb{P}[A]$ imply $\mathbb{P}[B|A] > \mathbb{P}[B]$?

2. We have three normal 6-sided dice and one die with three 1s and three 2s. We pick one of the dice uniformly at random and roll it.
(a) What is the probability of rolling a 1?
(b) If the outcome was 1, what is the probability that we picked a normal die?

3. Peter gets a lot of emails, but 80% of them are spam. His spam filter correctly flags 90% of the spam, but it also flags 5% of the regular emails as spam.
(a) What percentage of the emails will be marked as spam?
(b) What percentage of the emails marked as spam are actually not spam?
(c) What percentage of the emails not marked as spam are actually spam?

4. Alice has n coins, Bob has $n+1$ coins. They both flip all their coins and count the number of times they get heads. What is the probability that Bob gets more heads than Alice? (See the hints at the end for help.)

Bonus problems

5. (Simpson's paradox) In this problem, we will have two kinds of sweets: tasty yellow sweets and disgusting green sweets. However, we pick the sweets without looking (or we are colour-blind). The sweets are in four containers: a red jar, a black jar, a red bag and a black bag. Decide whether the following strange phenomenon can happen:

- The likelihood or getting a tasty sweet is greater if we take it out of the the red bag than if we take it out of the black jar.
- The likelihood or getting a tasty sweet is greater if we take it out of the red jar than if we take it out of the black bag.
- Now we transfer all the candies from red bag into the red jar, and all candies from the black bag into the black jar. After this operation, we are more likely to pull a tasty sweet if we reach into the black jar than if we reach into the red jar.

(See the hints at the end for help.)

6. (prosecutor's fallacy) Mrs. C's two children died shortly after birth. She is charged with double murder. The prosecutor argues as follows: The probability of sudden infant death syndrome is $\frac{1}{8500}$. So the probability that sudden infant death syndrome happens twice in a row is $\frac{1}{8500^2}$. Hence the probability that Mrs. C is innocent is $\frac{1}{8500^2}$, which is very low.

Formulate the prosecutor's arguments in the language of probability and find two errors in them.

7. There are m white and n black balls in a box. Two players take turns taking balls out of the box, the first one to take out a white ball loses. Describe an algorithm for computing the probability $p(m,n)$ that the first player loses. (See the hints at the end for help.)

More practice problems

8. The logical formula $A \Rightarrow B$ is equivalent to $\neg B \Rightarrow \neg A$. We will look at analogies involving probability.

(a) Show that if $\mathbb{P}[B|A] = 1$, then $\mathbb{P}[A^c|B^c] = 1$.

(b) Show, however, that it is possible to have $\mathbb{P}[B|A] \geq 0.9$ but $\mathbb{P}[A^c|B^c] = 0$.

9. We have a hat with 4 white, 4 black and 2 red balls. We will pull out 3 balls, one after another (without putting them back). What is the probability that the third of these balls is red?

10. You and your opponent are provided with three custom six-sided dice: die A has faces $1, 1, 5, 5, 9, 9$, die B has faces $2, 2, 6, 6, 7, 7$ and die C has faces $3, 3, 4, 4, 8, 8$. You each pick a die and roll it, whoever gets the higher number wins. Should you pick your die before or after your opponent?

Hint for problem 4: consider a scenario where Bob puts one coin aside and counts all the others, and only then will he consider the last one.

Hint for problem 5: given a fraction with a large numerator and a large denominator, adding small numbers to both has little influence on the value of the fraction.

Hint for problem 7: construct a recurrence formula, i.e. a formula for $p(m, n)$ using $p(m, n')$ for $n' \leq n$.
