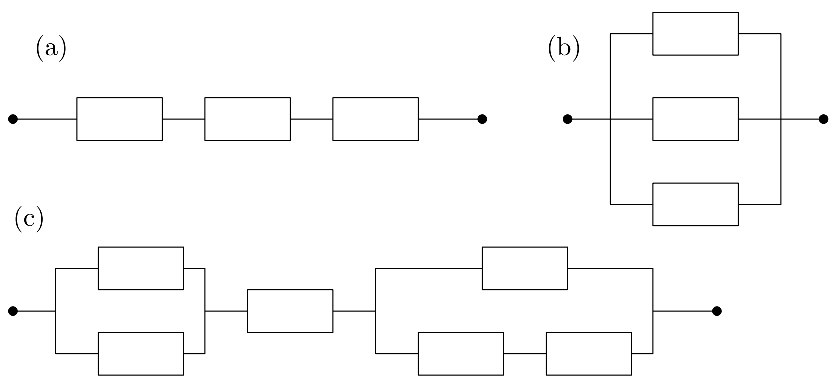
Problem set for Probability and Statistics 1 — 23/25 February 2026

Warm-up

1. Given a biased coin that comes up heads with some probability p, can we use it to simulate an unbiased coin toss? You can not pick the value of p and you do not know the value, you only know that $0 < p < 1$. However, you are allowed to toss the coin multiple times.

2. Each rectangle in the figure is a component that can fail with probability p in which case it does not conduct electricity. What is the probability that the current still flows between the two dots?



3. In a class of 5 students, a nursery teacher has 5 pencil boxes, each labelled with a different student's name. The teacher randomly distributes one pencil box to each student. What is the probability that at least one student receives the pencil box with their own name?

Conditional probability

4. A student is taking a multiple-choice exam in which each question has 5 possible answers, exactly one of which is correct. If the student knows the answer, she selects the correct answer. Otherwise, she chooses one option at random from the 5 possible answers. Suppose the student knows the answer to 70% of the questions.

(a) What is the probability that on a given question the student gets the correct answer?

(b) If the student gets the correct answer to a question, what is the probability that she knows the answer?

5. Toss two coins: a dime and a nickel. Each of them comes up heads with probability p.

(a) What is the probability that both come up heads if the dime comes up heads?

(b) What is the probability that both come up heads if we know that at least one of them comes up heads?

Is it clear without counting which probability is greater?

More practice problems

6. A six-sided die is rolled twice. Let A be the event "the first roll is either 1, 2 or 3", let B be the event "the first roll is either 3, 4 or 5" and let C be the event "the sum of the two rolls is 9". Show that $\mathbb{P}[A \cap B \cap C] = \mathbb{P}[A]\mathbb{P}[B]\mathbb{P}[C]$ but the events A, B, C are not independent.

7. We flip a coin and if it lands tails, we win the coin. We get a new coin for a new toss. We repeat the process until we get heads for the first time, then the game stops. We get all the coins that landed tails. (What is the probability that we get k coins?) After this we gather all the coins that we collected and toss them again, all at once. If all of them land tails, we can keep all of them. What is the probability of this happening?

8. There are four dead batteries in a box of 100.

(a) What is the probability that if we pick three of the batteries at random for the headlamp, it will work? (They all have to be live.)

(b) What is the probability that at least two out of three are live?

(c) We pull out a random battery three times, measure it, find it to be live each time and throw it back in. What is the probability of that happening?

(Without working out the numbers: is probability (a) higher or (c)?)

Bonus problems

9. (Bertrand's paradox, method 3) When choosing two points on a circle, by rotating the circle we can suppose without loss of generality that the chord connecting the two points is perpendicular to the radius pointing straight up and intersects with it. By considering the intersection point, argue that $\frac{1}{2}$ is a reasonable answer to "what is the probability that the two points selected are at least $120°$ apart?".

10. There are two envelopes on a table and we know that each envelope contains some (non-zero, integer) number of 1000 Kč notes, and the amount of money in the two envelopes is different. We are allowed to open one envelope and then decide whether to keep that one or to switch to the other. If we want to get the envelope with the higher amount, can we achieve this with probability greater than $\frac{1}{2}$? (See the hints at the end for help.)

11. There are a black and b white balls in the urn. We draw balls from it one by one (without returning them). What is the probability that the first ball drawn is black? Second, third, ...?

12. We have k containers, each containing a white and b black balls. We pick a random ball from the first urn and throw it into the second. Then we pick a random ball from the second urn, throw it into the third, etc. What is the probability of getting a white ball out of the last container?

Hints for the envelope problem: Pull out a coin and flip until it lands heads. Let X be the total number of flips (including the last one, i.e. $X \geq 1$.) If our envelope contains k notes, we keep the envelope if $X < k$.

What is the probability of getting an envelope with a higher amount?
