Abstracts of KAM-DIMATIA Series Year 2010

2010-955 R. J. Kang, J.-S. Sereni, and Matěj Stehlík
Every plane graph of maximum degree 8 has an edge-face 9-colouring

An edge-face colouring of a plane graph with edge set E and face set F is a colouring of the elements of $E \cup F$ such that adjacent or incident elements receive different colours. Borodin proved that every plane graph of maximum degree $\Delta \geq 10$ can be edge-face coloured with $\Delta + 1$ colours. Borodin's bound was recently extended to the case where $\Delta = 9$. In this paper we extend it to the case $\Delta = 8$.

2010-956 P. Kolman, B. Lidický, and J.-S. Sereni
On Minimum Fair Odd Cycle Transversal

In edge deletion problems, we are given a graph G and a graph property π and the task is to find a subset of edges the deletion of which results in a subgraph of G satisfying the property π. Typically the objective is to minimize the total number of deleted edges while in less common fair versions the objective is to minimize the maximum number of edges removed from a single vertex. We focus on the minimum fair odd cycle transversal (OCT) problem where the task is to make the graph bipartite; the problem is closely related to improper colorings of graphs. Though the classical version of the problem was diligently studied, the minimum fair version brings new challenges. We describe a $\Theta(\sqrt{n})$-approximation algorithm for general graphs and an exact polynomial time algorithm for graphs of bounded treewidth. Though there are several general frameworks (e.g., MSOL) for dealing with optimization problems on graphs of bounded treewidth, the minimum fair OCT does not seem to fit into any of them. Analogous results are proved for minimum fair cut problem.

2010-957 Jan Hubička and Jaroslav Nešetřil
Some examples of universal and generic partial orders

We survey structures endowed with natural partial orderings and prove their universality. These partial orders include partial orders on set of words, partial orders formed by geometric objects, grammars, polynomials and homomorphism order for various combinatorial objects.

2010-958 Petr Škoda
Computability of Width of Submodular Partition Functions

The notion of submodular partition functions generalizes many of well-known tree decompositions of graphs. For fixed k, there are polynomial-time algorithms to determine whether a graph has tree-width, branch-width, etc. at most k. Contrary to these results, we show that there is no sub-exponential algorithm for determining whether the width of a given submodular partition function is at most two. On the other hand, we show that for a subclass of submodular partition functions, which contains tree-width, there exists a polynomial-time algorithm that decides whether the width is at most k.

2010-959 Jiří Fink (ed.)
Midsummer Combinatorial Workshop 2009

The 15. Prague Midsummer Combinatorial Workshop was held from July 27th to July 31st 2009.

2010-960 M. Loebl and H. Teimoori
Bass' Identity and The Coin Arrangements Lemma

Lemma on coin arrangements is an important trick in Sherman's proof of Feynman conjecture on two dimensional Ising model. Here, we show that the coin arrangements lemma is indeed equivalent to the Bass' identity.

2010-961 Hossein Teimoori Faal
Determinants, Lyndon Covers and The Coin Arrangements Lemma

In this paper we first give a multiset version of the well-known graph theoretical interpretation of determinant of a matrix as a signed weighted sum over the cycle covers of the associated digraph of the matrix. Then, as a direct consequence of this new result, we give a multivariate generalization of the coin arrangements lemma.

2010-962 H. Khodakarami and H. Teimoori
Khayyam-Pascal Determinantal Arrays, Star of David Rule and Log-Concavity

In this paper we develop a new geometric method to answer the log-concavity questions related to a nice class of combinatorial sequences arising from the Khayyam-Pascal triangle.

2010-963 M. Hladík, and M. Černý
Interval regression by tolerance analysis approach

In interval regression analysis, we are given crisp or interval data and we are to determine appropriate interval regression parameters. There exist different methods for dealing with this problem; many of them possess the property that some of the resulting regression parameters are crisp. This property is undesirable in a variety of applications. To overcome this draw-back we propose a method motivated by tolerance analysis in linear systems. This method is not only computationally very cheap, but also yields intervals for regression parameters the widths of which are proportional to an in-advance given vector of parameters. For example, one choice of this vector allows to control relative tolerances and another leads to absolute tolerances. We show how the basic method, formulated for the model of crisp input – crisp output data, can be extended to interval data. For the interval-valued case we propose several formulations for the solution concept. We illustrate our approach by examples.

2010-964 Zuzka Safernová, Jan Volec (eds.)
Spring School on Combinatorics 2010

This year the Spring School is organized in Borová Lada, a mountain village in Šumava hills with a great variety of possibilities for outdoor activities like snow-shoe hiking or cross-country skiing.

2010-965 Bernhard Banaschewski and Aleš Pultr
Approximate maps, filter monad, and a representation of localic maps

A covariant representation of the category of locales by approximate maps (mimicking a natural representation of continuous maps between spaces in which one approximates points by small open sets) is constructed. It is shown that it can be given a Kleisli shape, as a part of a more general Kleisli representation of meet preserving maps. Also, we present the spectrum adjunction in this approximation setting.

2010-966 Jiří Fink
Towards a theory of Ground state uniqueness

We study groundstates in a generalization of Edwards-Anderson Ising model. We develop abstract discreate mathematical theory of incongruency called XOR-system.

2010-967 Richard N. Ball and Aleš Pultr
Quotients and colimits of κ-quantales

Let κ**Qnt** be the category of of κ-quantales, quantales closed under κ-joins in which the monoid identity is the largest element. (κ is an infinite regular cardinal.) Although the lack of lattice completeness in this setting would seem to mitigate against the techniques which lend themselves so readily to the calculation of frame quotients, we show how to easily compute κ**Qnt** quotients by applying generalizations of the frame techniques to suitable extensions of this category.

The second major tool in the analysis is the free κ-quantale over a λ-quantale, $\kappa \geq \lambda$. Surprisingly, these can be characterized intrinsically, and the generating sub-κ-quantale can even be identified. The result that the λ-free κ-quantales coincide with the λ-coherent κ-quantales directly generalizes Madden's corresponding result for κ-frames.

These tools permit a direct and intuitive construction of κ**Qnt** colimits. We provide two applications: an intrinsic characterization of κ**Qnt** colimits, and of free (over sets) κ-quantales. The latter is a direct generalization of Whitman's condition for distributive lattices.

2010-968 Martin Klazar
ANALYTIC AND COMBINATORIAL NUMBER THEORY I

These are lecture notes for the summer semester 2008 of the course *Analytic and combinatorial number theory* (NDMI045, *Analytická a kombinatorická teorie čísel*) which I have been teaching on the Faculty of Mathematics and Physics of the Charles University in Prague. The first booklet covers four major theorems, three on the distribution of prime numbers and one from additive combinatorics: Dirichlet's theorem on prime numbers in arithmetic progression, the Prime Number Theorem, Shnirel'man's theorem on sums of primes, and Roth's theorem on 3-term arithmetic progressions. Actually,

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2010-969 Martin Klazar
ANALYTIC AND COMBINATORIAL NUMBER THEORY II

These are lecture notes for the summer semester 2010 of the course *Analytic and combinatorial number theory* (NDMI045, *Analytická a kombinatorická teorie čísel*) which I have been teaching on the Faculty of Mathematics and Physics of the Charles University in Prague. In the second booklet we learn the theorems due to Thue (finiteness of solution set of Thue equation), Dirichlet (infinitude of primes in arithmetic progression) and Gel'fond and Schneider (transcendence of α^β for algebraic α and β).

2010-970 Jiří Fink
Doctoral Thesis Probabilistic Methodsin Discrete Applied Mathematics

One of the basic streams of modern statistical physics is an effort to understand the frustration and chaos. The basic model to study these phenomena is the finite dimensional Edwards-Anderson Ising model. We present a generalization of this model. We study set systems which are closed under symmetric differences. We show that the important question whether a groundstate in Ising model is unique can be studied in these set systems.

Kreweras' conjecture asserts that any perfect matching of the n-dimensional hypercube Q_n can be extended to a Hamiltonian cycle. We prove this conjecture.

The *matching graph* $\mathcal{M}(G)$ of a graph G has a vertex set of all perfect matchings of G, with two vertices being adjacent whenever the union of the corresponding perfect matchings forms a Hamiltonian cycle. We prove that the matching graph $\mathcal{M}(Q_n)$ is bipartite and connected for $n \geq 4$. This proves Kreweras' conjecture that the graph M_n is connected, where M_n is obtained from $\mathcal{M}(Q_n)$ by contracting all vertices of $\mathcal{M}(Q_n)$ which correspond to isomorphic perfect matchings.

A fault-free path in Q_n with f faulty vertices is said to be *long* if it has length at least $2^n - 2f - 2$. Similarly, a fault-free cycle in Q_n is long if it has length at least $2^n - 2f$. If all faulty vertices are from the same bipartite class of Q_n, such length is the best possible.

We show that for every set of at most $2n - 4$ faulty vertices in Q_n and every two fault-free vertices u and v satisfying a simple necessary condition on neighbors of u and v, there exists a long fault-free path between u and v. This number of faulty vertices is tight and improves the previously known results. We also consider much weaker condition of neighbors of u and v. We prove that for every set of at most $(n^2 + n - 4)/4$ faulty vertices of Q_n, there exists a long fault-free path between any two vertices such that each of them has at most 3 faulty neighbors.

Let $f(n)$ be the maximum integer such that for every set of at most $f(n)$ faulty vertices of Q_n, there exists a long fault-free cycle. Castañeda and Gotchev conjectured that $f(n) = \binom{n}{2} - 2$. First, we fount an elegant proof that $f(n) \geq n^2/10 + n/2 + 1$ for $n \geq 15$ which was the first known quadratic lower bound. Later, we proved this conjecture using new technique of potentials which we introduced.

2010-971 Martin Klazar
Diophantine equation $ax^n - by^n = c$. I

This exposition is a first draft of a fragment of a future book on number theory. We explain on the Diophantine equation $ax^n - by^n = c$ ($a, b, n \in \mathbb{N}$, $n \geq 3$, $c \in \mathbb{Z}$ and $c \neq 0$) the hypergeometric method as developed by Thue, Siegel and Baker. We show that (i) the equation has only finitely many solutions $x, y \in \mathbb{Z}$ (Thue's non-effective result), (ii) for ab sufficiently large depending on n and c there is at most one solution $x, y \in \mathbb{N}$ (Siegel's result) and (iii) for a, b, n in a certain range there is an explicit bound on the sizes of solutions and thus an algorithm for determining them (Baker's effective result). We also discuss Thue's effective result from 1918.

2010-972 J. Picado and A. Pultr
On strong inclusions and asymmetric proximities in frames

The strong inclusion, a specific type of subrelation of the order of a lattice with pseudocomplements, has been used in the concrete case of the lattice of open sets in topology for an expedient definition of proximity, and allowed for a natural point-free extension of this concept. A modification of a strong inclusion for biframes then provided a point-free model also for the non-symmetric variant. In this paper we show that a strong inclusion can be non-symmetrically modified to work directly on frames, without prior assumption of a biframe structure. The category of quasi-proximal frames thus obtained is shown to be concretely isomorphic with the biframe based one, and shown to be related to that of quasi-uniform frames in a full analogy with the symmetric case.

2010-973 Milan Hladík
How to determine basis stability in interval linear programming

Interval linear programming (ILP) was introduced in order to deal with linear programming problems with uncertainties that are modelled by ranges of admissible values. Basic tasks in ILP such as calculating the optimal value bounds or set of all possible solutions may be computationally very expensive. However, if some basis stability criterion holds true then the problems becomes much more easy to solve. In this paper, we propose a method for testing basis stability. Even though the method is exponential in the worst case (not surprisingly), it is fast in many cases.

2010-974 D. Garijo, A. Goodall, and J. Nešetřil
Distinguishing graphs by their left and right homomorphism profiles

We introduce a new property of graphs called 'q-state Potts uniqueness' and relate it to chromatic and Tutte uniqueness, and also to 'chromatic–flow uniqueness', recently studied by Duan, Wu and Yu.

We establish for which edge-weighted graphs H homomorphism functions from multigraphs G to H are specializations of the Tutte polynomial of G, in particular answering a question of Freedman, Lovász and Schrijver. We also determine for which edge-weighted graphs H homomorphism functions from multigraphs G to H are specializations of the 'edge elimination polynomial' of Averbouch, Godlin and Makowsky and the 'induced subgraph polynomial' of Tittmann, Averbouch and Makowsky.

Unifying the study of these and related problems is the notion of the left and right homomorphism profiles of a graph.

2010-975 Z. Dvořák, K. Kawarabayashi, and D. Král'
Packing six T-joins in plane graphs

Let G be a plane graph and T an even subset of its vertices. It has been conjectured that if all T-cuts of G have the same parity and the size of every T-cut is at least k, then G contains k edge-disjoint T-joins. The case $k = 3$ is equivalent to the Four Color Theorem, and the cases $k = 4$, which was conjectured by Seymour, and $k = 5$ were proved by Guenin. We settle the next open case $k = 6$.

2010-976 J. Picado and A. Pultr
Notes on entourages and localic groups

The relation between the cover (Tukey type) uniformities and the entourage (Weil type) ones, in the point-free context, is studied and a transparent translation is presented. In particular the natural uniformities on localic groups are discussed, and the uniformity of localic group homomorphisms is proved.

2010-977 Z. Dvořák, D. Král', and R. Thomas
Three-coloring triangle-free graphs on surfaces I. Extending a coloring to a disk with one triangle

Let G be a plane graph with with exactly one triangle T and all other cycles of length at least 5, and let C be a facial cycle of G of length at most six. We prove that a 3-coloring of C does not extend to a 3-coloring of G if and only if C has length exactly six and there is a color x such that either G has an edge joining two vertices of C colored x, or T is disjoint from C and every vertex of T is adjacent to a vertex of C colored x. This is a lemma to be used in a future paper of this series.

2010-978 Z. Dvořák and B. Mohar
Spectrally degenerate graphs: Hereditary case

It is well known that the spectral radius of a tree whose maximum degree is Δ cannot exceed $2\sqrt{\Delta - 1}$. Similar upper bound holds for arbitrary planar graphs, whose spectral radius cannot exceed $\sqrt{8\Delta} + 10$, and more generally, for all d-degenerate graphs, where the corresponding upper bound is $\sqrt{4d\Delta}$. Following this, we say that a graph G is *spectrally d-degenerate* if every subgraph H of G has spectral radius at most $\sqrt{d\Delta(H)}$. In this paper we derive a rough converse of the above-mentioned results by proving that each spectrally d-degenerate graph G contains a vertex whose degree is at most $4d \log_2(\Delta(G)/d)$ (if $\Delta(G) \geq 2d$). It is shown that the dependence on Δ in this upper bound cannot be eliminated, as long as the dependence on d is subexponential. It is also proved that the problem of deciding if a graph is spectrally d-degenerate is co-NP-complete.

2010-979 F. Kardoš, D. Král', and J. Volec
Fractional colorings of cubic graphs with large girth

We show that every (sub)cubic n-vertex graph with sufficiently large girth has fractional chromatic number at most 2.2978 which implies that it contains an independent set of size at least $0.4352n$. Our bound on the independence number is valid to random cubic graphs as well as it improves existing lower bounds on the maximum cut in cubic graphs with large girth.

2010-980 Milan Hladík
Complexity of necessary efficiency in interval LP and MOLP

We present some complexity results on testing necessary efficiency in interval multiobjective linear programming. Supposing that objective function coefficients perturb within prescribed intervals, a feasible point x^* is called necessarily efficient if it is efficient for all instances of interval data. We show that the problem of testing necessary efficiency is co-NP-complete even for only one objective. Provided that x^* is non-degenerate basic solution, the problem is polynomially solvable for one objective, but remains NP-hard in the general case. Some open problems are mentioned at the end of the paper.

2010-981 Milan Hladík

Interval linear programming: A survey

Uncertainty is a common phenomenon in practice. Due to measurement errors we can hardly expect precise values in real-life linear programming problems. Using estimated quantities may lead to unsatisfactory results, so inexactness must be taken into account. Uncertainty can be handled in various manners, e.g. by stochastic programming, interval analysis or fuzzy numbers; each of them has some pros and cons. In this paper, we suppose that we are given lower and upper bounds on the quantities, and the quantities may perturb independently and simultaneously within these bounds. In this model we investigate the problems of optimal value range, basis stability, optimal solutions enclosures, duality etc. Complexity issues are discussed, too; some tasks are polynomially solvable while another are NP-hard.

This approach is more general and powerful than the standard sensitivity analysis. In sensitivity analysis, we consider variations of only one parameter, which is very restrictive. On the other hand, interval analysis based approach enables to handle simultaneously all required parameters. We present a brief exposition of the known results with new insights, and close the survey by some challenging problems.

2010-982 D. Král', O. Serra, and L. Vena

On the Removal Lemma for linear systems over Abelian groups

In this paper we present an extension of the removal lemma to integer linear systems over abelian groups. We prove that, if the k–determinantal of an integer $(k \times m)$ matrix A is coprime with the order n of a group G and the number of solutions of the system $Ax = b$ with $x_1 \in X_1, \ldots, x_m \in X_m$ is $o(n^{m-k})$, then we can eliminate $o(n)$ elements in each set to remove all these solutions.

2010-983 Milan Hladík

Enclosures for the solution set of parametric interval linear systems

We investigate parametric interval linear systems of equations. The main result is a generalization of the Bauer–Skeel and Hansen–Bliek–Rohn bounds for this case, comparing and refinement of both. We show that not only the latter bounds are not provably better, but they are sometimes too pessimistic. The presented form of both methods is suitable for combining both of them into one to get a more efficient algorithm. Some numerical experiments are carried out to illustrate performances of the methods.

2010-984 D. Garijo, A. Goodall, and J. Nešetřil
Contractors for flows

We answer a question raised by Lovász and B. Szegedy [Contractors and connectors in graph algebras, *J. Graph Theory* 60:1 (2009)] asking for a contractor for the graph parameter counting the number of B-flows of a graph, where B is a subset of a finite Abelian group closed under inverses. We prove our main result using the duality between flows and tensions and finite Fourier analysis. We exhibit several examples of contractors for B-flows, which are of interest in relation to the family of B-flow conjectures formulated by Tutte, Fulkerson, Jaeger, and others.

2010-985 T. Dvořák, J. Fink, P. Gregor, V. Koubek, and T. Radzik
Efficient connectivity testing of hypercubic networks with faults

Given a connected graph G and a set F of faulty vertices of G, let $G - F$ be the graph obtained from G by deletion of all vertices of F and edges incident with them. Is there an algorithm, whose running time may be bounded by a polynomial function of $|F|$ and $\log |V(G)|$, which decides whether $G - F$ is still connected? Even though the answer to this question is negative in general, we describe an algorithm which resolves this problem for the n-dimensional hypercube in time $O(|F|n^3)$. Furthermore, we sketch a more general algorithm that is efficient for graph classes with good vertex expansion properties.

2010-986 T. Dvořák, J. Fink, P. Gregor, V. Koubek
Gray codes with bounded weights

Given a set H of binary vectors of length n, is there a cyclic listing of H so that every two successive vectors differ in a single coordinate? The problem of existence of such a listing, which is called a *cyclic Gray code* of H, is known to be NP-complete in general. The goal of this paper is therefore to specify boundaries between its intractability and polynomial decidability.

For that purpose, we consider a restriction when the vectors of H are of a bounded weight. A *weight* of a vector $u \in \{0,1\}^n$ is the number of 1's in u. We show that if every vertex of H has weight k or $k+1$, our problem is polynomial for $k \leq 1$ and NP-complete for $k \geq 2$. Furthermore, if $k = 2$ and for every $i \in [n]$ there are at most m vectors of H of weight two having one in the i-th coordinate, then the problem becomes polynomial for $m \leq 3$ and NP-complete for $m \geq 13$.

The following complementary problem is also known to be NP-hard: given an $F \subseteq \{0,1\}^n$, which now plays the role of a set of faults to be avoided, is there a cyclic Gray code of $\{0,1\}^n \setminus F$? We show that if every vertex of F has weight at most k, the problem is polynomial for $k \leq 2$ and NP-hard for $k \geq 5$. It follows that there is a function $f(n) = \Theta(n^4)$ such that the existence of a cyclic Gray code of $\{0,1\}^n \setminus F$ for a given set $F \subseteq \{0,1\}^n$ of size at most $f(n)$ is NP-hard.

In addition, we study the cases when the Gray code does not have to be cyclic, and moreover, when the first and the last vectors of the code are prescribed. For these two modifications, all NP-hardness and NP-completeness results hold as well.

2010-987 Z. Dvořák, B. Mohar, and R. Šámal
Star chromatic index

The star chromatic index $\chi'_s(G)$ of a graph G is the minimum number of colors needed to properly color the edges of the graph so that no path or cycle of length four is bi-colored. We obtain a near-linear upper bound in terms of the maximum degree $\Delta = \Delta(G)$. Our best lower bound on χ'_s in terms of Δ is $2\Delta(1 + o(1))$ valid for complete graphs. We also consider the special case of cubic graphs, for which it is shown that the star chromatic index lies between 4 and 7. The proofs involve a variety of notions from other branches of mathematics and may therefore be of certain independent interest.

2010-988 L. Esperet, F. Kardoš, A. King, D. Král', and S. Norine
Exponentially many perfect matchings in cubic graphs

We show that every cubic bridgeless graph G has at least $2^{|V(G)|/3656}$ perfect matchings. This confirms an old conjecture of Lovász and Plummer.

2010-989 M. Hladík and S. Sitarz

Maximal and supremal tolerances in multiobjective linear programming

Let a multiobjective linear programming problem and any efficient solution be given. Tolerance analysis aims to compute interval tolerances for (possibly all) objective function coefficients such that the efficient solution remains efficient for any perturbation of the coefficients within the computed intervals. The known methods yield tolerances that are not optimal. In this paper, we propose a method for calculating the supremal tolerance (the maximal one needn't exist). The method is exponential in the worst case. We show that the problem of determining the maximal/supremal tolerance is NP-hard, so an efficient procedure is not likely to exist.

2010-990 R. Šámal, R. Stolař, T. Valla

Complexity of the cop & robber guarding game

The guarding game is a game in which a set of cops has to guard a region in a digraph against a robber. The robber and the cops are placed on vertices of the graph; they take turns in moving to adjacent vertices (or staying). The goal of the robber is to enter the guarded region at a vertex with no cop on it. The problem is to find the minimum number of cops needed to prevent the robber from entering the guarded region. The problem is highly nontrivial even for very simple graphs – when the robber moves in a tree, then the decision version of the problem is NP-complete. Furthermore, if the robber is moving in a DAG, the problem becomes PSPACE-complete. This was the work of Fomin, Golovach, Hall, Mihalák, Vicari and Widmayer. We solve the question asked by Golovach and we show that if the graph is an arbitrary directed or undirected graph, the problem becomes ETIME-complete.

2010-991 K. Junosza-Szaniawski and P. Rzewski

On the number of 2-packings in a connected graph

By a 2-packing in a graph we mean a subset of its vertex set, in which all the vertices are in distance at least 3 from each other.

The question about the maximum number of 2-packings in a graph is strongly related to the problem of $L(2,1)$-labeling of graphs.

In this paper we find new asymptotic upper and lower bounds on the maximum number of 2-packings in a connected graph on n vertices. The bounds are $O(1.5399..^n)$ and $\Omega(1.4970..^n)$, respectively.

Moreover, we present a lower bound on the number of k-element 2-packings in a connected graph, which is $\max(\binom{n-2k+2}{k}, (k+1)\binom{\lfloor\frac{n-1}{2}\rfloor}{k})$.

2010-992 K. Junosza-Szaniawski and P. Rzewski

On the Complexity of Exact Algorithm for $L(2,1)$-labeling of Graphs

$L(2,1)$-labeling is a graph coloring model inspired by a frequency assignment in telecommunication. It asks for such a labeling of vertices with nonnegative integers that adjacent vertices get labels that differ by at least 2 and vertices in distance 2 get different labels. It is known that for any $k \geq 4$ it is NP-complete to determine if a graph has a $L(2,1)$-labeling with no label greater than k.

In this paper we present a new bound on complexity of an algorithm for finding an optimal $L(2,1)$-labeling (i.e. an $L(2,1)$-labeling in which the largest label is the least possible). We improve the upper complexity bound of the algorithm from $O^*(3.5615..^n)$ to $O^*(3.2360..^n)$. Moreover, we establish a lower complexity bound of the presented algorithm, which is $\Omega^*(3.0731..^n)$.
