# Abstracts of KAM-DIMATIA Series Year 2009

Martin Kupec (ed.)

2009-907 V. Jungić, T. Kaiser, and D. Král'
**A note on edge-colourings avoiding rainbow $K_4$ and monochromatic $K_m$**

We study the mixed Ramsey number $maxR(n, K_m, K_r)$, defined as the maximum number of colours in an edge-colouring of the complete graph $K_n$, such that $K_n$ has no monochromatic complete subgraph on $m$ vertices and no rainbow complete subgraph on $r$ vertices. Improving an upper bound of Axenovich and Iverson, we show that $maxR(n, K_m, K_4) \leq n^{3/2}\sqrt{2m}$ for all $m \geq 3$. Further, we discuss a possible way to improve their lower bound on $maxR(n, K_4, K_4)$ based on incidence graphs of finite projective planes.

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2009-908 T. Müller, A. Pór, and J. Sereni
**Graphs with four boundary vertices**

A vertex $v$ of a graph $G$ is a *boundary vertex* if there exists a vertex $u$ such that the distance in $G$ from $u$ to $v$ is at least the distance from $u$ to any neighbour of $v$. We give a full description of all graphs that have exactly four boundary vertices, which answers a question of Hasegawa and Saito. To this end, we introduce the concept of frame of a graph. It allows us to construct, for every positive integer $b$ and every possible "distance-vector" between $b$ points, a graph $G$ with exactly $b$ boundary vertices such that every graph with $b$ boundary vertices and the same distance-vector between them is an induced subgraph of $G$.

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2009-909 J. Nešetřil, P. Ossona de Mendez, and D. Wood
**Characterisations and Examples of Graph Classes with Bounded Expansion**

Classes with bounded expansion, which generalise classes that exclude a topological minor, have recently been introduced by Nešetřil and Ossona de Mendez. These classes are defined by the fact that the maximum average degree of a shallow minor of a graph in the class is bounded by a function of the depth of the shallow minor. Several linear-time algorithms are known for bounded expansion classes (such as subgraph isomorphism testing), and they allow restricted homomorphism dualities, amongst other desirable properties.

In this paper we establish two new characterisations of bounded expansion classes, one in terms of so-called topological parameters, the other in terms of controlling dense parts. The latter characterisation is then used to show that the notion of bounded expansion is compatible with Erdös-Rényi model of random graphs with constant average degree. In particular, we prove that for every fixed $d > 0$, there exists a class with bounded expansion, such that a random graph of order $n$ and edge probability $d/n$ asymptotically almost surely belongs to the class.

We then present several new examples of classes with bounded expansion that do not exclude some topological minor, and appear naturally in the context of graph drawing or graph colouring. In particular, we prove that the following classes have bounded expansion: graphs that can be drawn in the plane with a bounded number of crossings per edge, graphs with bounded stack number, graphs with bounded queue number, and graphs with bounded non-repetitive chromatic number. We also prove that graphs with 'linear' crossing number are contained in a topologically-closed class, while graphs with bounded crossing number are contained in a minor-closed class.

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2009-910 Milan Hladík

**Generalized linear fractional programming under interval uncertainty**

Data in many real-life problems suffer from inexactness. Herein we assume that we are given some intervals in which the data can simultaneously perturb. We consider a generalized linear fractional programming problem with interval data and present an efficient method for computing the range of optimal values. We consider also the inverse problem: How much can data of a real generalized linear fractional program vary such that the optimal values do not exceed some prescribed bounds. We illustrate the approach on a simple von Neumann economic growth model.

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2009-911 J. Böttcher, J. Hladký, and D. Piguet
**The tripartite Ramsey number for trees**

We prove that for all $\varepsilon > 0$ there are $\alpha > 0$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ the following holds. For any two-colouring of the edges of $K_{n,n,n}$ one colour contains copies of all trees $T$ of order $t \leq (3-\varepsilon)n/2$ and with maximum degree $\Delta(T) \leq n^{\alpha}$. This confirms a conjecture of Schelp.

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2009-912 B. Banaschewski and A. Pultr
**Pointfree aspects of the $T_D$ axiom of classical topology**

Abstraction from the condition defining $T_D$-spaces leads to the following notion in an arbitrary frame $L$: a filter $F$ in $L$ is called *slicing* if it is prime and there exist $a, b \in L$ such that $a \notin F$, $b \in F$, and $a$ is covered by $b$. This paper deals with various aspects of these slicing filters. As a first step, we present several results about the original $T_D$ condition. Next, concerning slicing filters, we show they are completely prime and characterize them in various ways. In addition, we prove for the frames $\mathfrak{O}X$ of open subsets of a space $X$ that every slicing filter is an open neighbourhood filter $\mathcal{U}(x)$ and $X$ is $T_D$ iff every $\mathcal{U}(x)$ is slicing. Further, for $\mathbf{Top}_D$ and $\mathbf{Frm}_D$ the categories of $T_D$ spaces and their continuous maps, and all frames and those homomorphisms whose associated spectral maps preserve the completely prime elements, respectively, we show that the usual contravariant functors between **Top** and **Frm** induce analogous functors here, providing a dual equivalence between $\mathbf{Top}_D$ and the subcategory of $\mathbf{Frm}_D$ given by the $T_D$-spatial frames (not coinciding with the spatial ones). In addition, we show that $\mathbf{Top}_D$ is mono-coreflective in a suitable subcategory of **Top**. Finally, we provide a comparison between $T_D$-separation and sobriety showing they may be viewed, in some sense, as mirror images of each other.

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2009-913 J. Maxová, P. Pavlíková, and D. Turzík
**On the complexity of cover-incomparability graphs of posets**

In this paper we show that the recognition problem for C-I graphs of posets is NP-complete. On the other hand, we prove that induced subgraphs of C-I graphs are exactly complements of comparability graphs, and hence the recognition problem for induced subgraphs of C-I graphs of posets is polynomial.

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2009-914 M. DeVos, A. Georgakopoulos, B. Mohar, and R. Šámal
**An Eberhard-like theorem for pentagons and heptagons**

Eberhard proved that for every sequence $(p_k), 3 \leq k \leq r, k \neq 5, 7$ of non-negative integers satisfying Euler's formula $\sum_{k\geq 3}(6-k)p_k = 12$, there are infinitely many values $p_6$ such that there exists a simple convex polyhedron having precisely $p_k$ faces of length $k$ for every $k \geq 3$, where $p_k = 0$ if $k > r$. In this paper we prove a similar statement when non-negative integers $p_k$ are given for $3 \leq k \leq r$, except for $k = 5$ and $k = 7$. We prove that there are infinitely many values $p_5, p_7$ such that there exists a simple convex polyhedron having precisely $p_k$ faces of length $k$ for every $k \geq 3$. We derive an extension to arbitrary closed surfaces, yielding maps of arbitrarily high face-width. Our proof suggests a general method for obtaining results of this kind.

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2009-915 J. Hladký and D. Král'
**Algebraic proof of Brooks' theorem**

We give a proof of Brooks' theorem as well as its list coloring extension using the algebraic method of Alon and Tarsi.

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2009-916 Jana Maxová (ed.)
**Midsummer Combinatorial Workshop 2008**

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2009-917 Jiří Matoušek
**Selected Mathematical and Algorithmic Applications of Linear Algebra**

Some years ago I started gathering nice applications of linear algebra. Here are some pieces from the collection. The applications belong mostly to the main fields of my mathematical interests—combinatorics, geometry, and computer science. Most of them are mathematical, in proving theorems, and some include clever ways of computing things, i.e., algorithms. The appearance of linear-algebraic methods is often unexpected.

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2009-918 Yared Nigussie
**Extended Gallai's Theorem**

Let $G$ and $H$ be graphs. We say $G$ is $H$-critical, if every proper subgraph of $G$ except $G$ itself is homomorphic to $H$. This generalizes the widely known concept of $k$-color-critical graphs, as they are the case $H = K_{k-1}$. In 1963 Gallai proved that the vertices of degree $k$ in a $K_k$-critical graph induce a subgraph whose blocks are either odd cycles or complete graphs. We generalize Gallai's Theorem for every $H$-critical graph, where $H = K_{k-2} + H'$, (the join of a complete graph $K_{k-2}$ with any graph $H'$). This answers one of the two unknown cases of a problem given in [J. Nešetřil, Y. Nigussie, Finite dualities and map-critical graphs on a fixed surface]. We also propose an open question, which may be a characterization of all graphs for which Gallai's Theorem holds.

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2009-919 P. Allen, J. Böttcher, and J. Hladký
**Filling the gap between Turán's theorem and Pósa's conjecture**

Much of extremal graph theory has concentrated either on finding very small subgraphs of a large graph (such as Turán's theorem) or on finding spanning subgraphs (such as Dirac's theorem or more recently the Pósa conjecture). Only a few results give conditions to obtain some intermediate-sized subgraph. We contend that this neglect is unjustified. In this paper we investigate minimum-degree conditions under which a graph $G$ contains squared paths and squared cycles of arbitrary specified lengths. We determine precise thresholds, assuming that the order of $G$ is large. This extends results of Fan and Kierstead [J. Combin. Theory Ser. B 63 (1995), 55–64] and of Komlós, Sarközy, and Szemerédi [Random Structures Algorithms 9 (1996), 193–211] concerning containment of a spanning squared paths and a spanning squared cycle, respectively.

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2009-920 Milan Hladík
**Interval valued bimatrix games**

Payoffs in (bimatrix) games are usually not known precisely, but it is often possible to determine lower and upper bounds on payoffs. Such interval valued bimatrix games are considered in this paper. There are many questions arising in this context. First, we discuss the problem of existence of an equilibrium being common for all instances of interval values. We show that this property is equivalent to solvability of a certain linear mixed integer system of equations and inequalities. Second, we characterize the set of all possible equilibria by mean of a linear mixed integer system.

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2009-921 J. Foniok, J. Nešetřil, A. Pultr, and C. Tardiff

**Dualities and dual pairs in Heyting algebras**

We extract the abstract core of finite homomorphism dualities using the techniques of Heyting algebras and (combinatorial) categories.

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2009-922 J.-S. Sereni and M. Stehlík

**Edge-face colouring of plane graphs with maximum degree nine**

An edge-face-colouring of a plane graph with edge set $E$ and face set $F$ is a colouring of the elements of $E \cup F$ so that adjacent or incident elements receive different colours. Borodin proved that every plane graph of maximum degree $\Delta \geq 10$ can be edge-face-coloured with $\Delta + 1$ colours. We extend Borodin's result to the case where $\Delta = 9$.

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2009-923 J. Hladký and M. Schacht

**Note on bipartite graph tilings**

Let $s < t$ be two fixed positive integers. We study what are the minimum degree conditions for a bipartite graph $G$, with both color classes of size $n = k(s + t)$, which ensure that $G$ has a $K_{s,t}$-factor. Exact result for large $n$ is given.

Our result extends the work of Zhao, who determined the minimum degree threshold which guarantees that a bipartite graph has a $K_{s,s}$-factor.

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2009-924 F. Havet, B. Reed, and J.-S. Sereni

**L(p,1)-labelling of graphs**

An $L(p, 1)$-labelling of a graph is a function $f$ from the vertex set to the positive integers such that $|f(x) - f(y)| \geq p$ if $\text{dist}(x, y) = 1$ and $|f(x) - f(y)| \geq 1$ if $\text{dist}(x, y) = 2$, where $\text{dist}(x, y)$ is the distance between the two vertices $x$ and $y$ in the graph. The *span* of an $L(p, 1)$-labelling $f$ is the difference between the largest and the smallest labels used by $f$ plus 1. In 1992, Griggs and Yeh conjectured that every graph with maximum degree $\Delta \geq 2$ has an $L(2, 1)$-labelling with span at most $\Delta^2 + 1$. We settle this conjecture for $\Delta$ sufficiently large. More generally, we show that for any positive integer $p$ there exists a constant $\Delta_p$ such that every graph with maximum degree $\Delta \geq \Delta_p$ has an $L(p, 1)$-labelling with span at most $\Delta^2 + 1$. This yields that, for each positive integer $p$, there is an integer $C_p$ such that every graph with maximum degree $\Delta$ has an $L(p,1)$-labelling with span at most $\Delta^2 + C_p$.

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2009-925 D. Král', P. Škoda, and J. Volec

**Domination number of cubic graphs with large girth**

We show that every $n$-vertex cubic graph with girth at least $g$ have domination number at most $0.299871n + O(n/g) < 3n/10 + O(n/g)$.

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2009-926 Louis Esperet

**Dynamic list coloring of bipartite graphs**

A *dynamic coloring* of a graph is a proper coloring of its vertices such that every vertex of degree more than one has at least two neighbors with distinct colors. The least number of colors in a dynamic coloring of $G$, denoted by $\chi_2(G)$, is called the *dynamic chromatic number* of $G$. The least integer $k$, such that if every vertex of $G$ is assigned a list of $k$ colors, then $G$ has a proper (resp. dynamic) coloring in which every vertex receives a color from its own list, is called the *choice number* of $G$, denoted $ch(G)$ (resp. the *dynamic choice number*, denoted $ch_2(G)$). It was recently conjectured [S. Akbari *et al.*, *On the list dynamic coloring of graphs*, Discrete Appl. Math. (2009)] that for any graph $G$, $ch_2(G) = \max(ch(G), \chi_2(G))$. In this short note we disprove this conjecture. We first give the example of a small planar bipartite graph $G$ with $ch(G) = \chi_2(G) = 3$ and $ch_2(G) = 4$. Then, for any integer $k \geq 5$, we construct a bipartite graph $G_k$ such that $ch(G_k) = \chi_2(G_k) = 3$ and $ch_2(G) \geq k$.

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2009-927 L. Esperet, J. Gimbel, and A. King

**Covering line graphs with equivalence relations**

An equivalence graph is a disjoint union of cliques, and the equivalence number $eq(G)$ of a graph $G$ is the minimum number of equivalence subgraphs needed to cover the edges of $G$. We consider the equivalence number of a line graph, giving improved upper and lower bounds: $\frac{1}{3}\log_2\log_2\chi(G) < eq(L(G)) \leq 2\log_2\log_2\chi(G) + 2$. This disproves a recent conjecture that $eq(L(G))$ is at most three for triangle-free $G$; indeed it can be arbitrarily large.

To bound $eq(L(G))$ we bound the closely-related invariant $\sigma(G)$, which is the minimum number of orientations of $G$ such that for any two edges

$e, f$ incident to some vertex $v$, both $e$ and $f$ are oriented out of $v$ in some orientation. When $G$ is triangle-free, $\sigma(G) = eq(L(G))$. We prove that even when $G$ is triangle-free, it is NP-complete to decide whether or not $\sigma(G) \leq 3$.

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2009-928 J. Nešetřil and P. Ossona de Mendez
**On Nowhere Dense Graphs**

A set $A$ of vertices of a graph $G$ is called $d$-scattered in $G$ if no two $d$-neighborhoods of (distinct) vertices of $A$ intersect. In other words, $A$ is $d$-scattered if no two distinct vertices of $A$ have distance at most $2d$. This notion was isolated in the context of finite model theory by Gurevich and recently it played a prominent role in the study of homomorphism preservation theorems for special classes of structures (such as minor closed families). This in turn led to the notions of wide, semiwide and quasi-wide classes of graphs. It has been proved previously that minor closed classes and classes of graphs with locally forbidden minors are examples of such classes and thus (relativised) homomorphism preservation theorem holds for them. In this paper we show that (more general) classes with bounded expansion and (newly defined) classes with bounded local expansion and even (very general) classes of nowhere dense graphs are quasi wide. This not only strictly generalizes the previous results and solves several open problems but it also provides new proofs. It appears that bounded expansion and nowhere dense classes are perhaps a proper setting for investigation of wide-type classes as in several instances we obtain a structural characterization. This also puts classes of bounded expansion in the new context and we are able to prove a trichotomy result which separates classes of graphs which are dense (somewhere dense), nowhere dense and finite. Our motivation stems from finite dualities. As a corollary we obtain that any homomorphism closed first order definable property restricted to a bounded expansion class is a duality.

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2009-929 Marek Tesař (ed.)
**Workshop on Coverings and Colorings 2009**

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2009-930 Milan Hladík
**Support set invariancy for interval bimatrix games**

Traditionally, game theory problems were considered for exact data and the decisions were based on known payoffs. However, this assumption is rarely true in practice. Uncertainty in measurement and imprecise information must be taken into account. Interval-based approach for handling such uncertainties assumes that one has lower and upper bounds on payoffs. Herein, interval bimatrix games are studied.

Especially, we focus on three kinds of support set invariancy. Support of a mixed strategy consists of that pure strategies having positive probabilities. Given an interval-valued bimatrix game and supports for both players, the question states as follows: Does every bimatrix game instance have an equilibrium with the prescribed support? The other two kinds of invariancies are slight modifications: Has every bimatrix game instance an equilibrium being a subset/superset of the prescribed support? It is difficult to answer these questions: the first case costs solving a large number of linear programs or mixed integer programs. For the remaining two cases a sufficient condition and a necessary condition is proposed, respectively.

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2009-931 J. Hubička and J. Nešetřil
**On characteristics of homomorphism and embedding universal graphs**

We relate the existence problem of universal objects to the properties of corresponding enriched categories (lifts and expansions). Particularly, extending earlier results, we prove that for every countable set $\mathcal{F}$ of finite structures there exists a (countably) universal structure $\mathbf{U}$ for the class $Forb_h(\mathcal{F})$ (of all countable structures omitting a homomorphism from all members of $\mathcal{F}$). In fact $\mathbf{U}$ is the shadow (reduct) of an ultrahomogeneous structure $\mathbf{U}'$ (which however, as we will show, cannot be expressed as $Forb_h(\mathcal{F}')$ for a countable set $\mathcal{F}$; this is in a sharp contrast to the case when $\mathcal{F}$ is finite). We also put the results of this paper, perhaps for the first time, in the context of homomorphism dualities and Constraint Satisfaction Problems.

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2009-932 J. Volec and Z. Safernová
**Spring School on Combinatorics 2009**

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2009-933 Jan Kynčl, Bernard Lidický, and Tomáš Vyskočil
**Irreversible 2-conversion set is NP-complete**

An irreversible $k$-threshold process is a process on a graph where vertices change color from white to black if they have at least $k$ black neighbors. An irreversible $k$-conversion set is a subset $S$ of vertices of a graph $G$ such that the irreversible $k$-threshold process changes all vertices of $G$ to black if $S$ is the initial set of black vertices. We show that deciding the existence of an irreversible 2-conversion set of a given size is NP-complete which answers a question of Dreyer and Roberts. Moreover, we show an optimal irreversible 3-conversion set for a toroidal grid, which simplifies constructions of Pike and Zou.

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2009-934 J. Hladký, D. Král', and S. Norine
**Counting flags in triangle-free digraphs**

Motivated by the Caccetta-Häggkvist Conjecture, we prove that every digraph on $n$ vertices with minimum outdegree $0.3465n$ contains an oriented triangle. This improves the bound of $0.3532n$ of Hamburger, Haxell and Kostochka. The main new tool we use in our proof is the theory of flag algebras developed recently by Razborov.

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2009-935 Martin Tancer
**Non-representability of finite projective planes by convex sets**

We prove that there is no $d$ such that all finite projective planes can be represented by convex sets in $\mathbb{R}^d$, answering a question of Alon, Kalai, Matoušek, and Meshulam. Here, if $\mathbb{P}$ is a projective plane with lines $\ell_1, \ldots, \ell_n$, a *representation of* $\mathbb{P}$ *by convex sets* in $\mathbb{R}^d$ is a collection of convex sets $C_1, \ldots, C_n \subseteq \mathbb{R}^d$ such that $C_{i_1}, C_{i_2} \ldots, C_{i_k}$ have a common point if and only if the corresponding lines $\ell_{i_1}, \ldots, \ell_{i_k}$ have a common point in $\mathbb{P}$. The proof combines a positive-fraction selection lemma of Pach with a result of Alon on "expansion" of finite projective planes. As a corollary, we show that for every $d$ there are 2-collapsible simplicial complexes that are not $d$-representable, strengthening a result of Matoušek and the author.

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2009-936 Martin Tancer
**$d$-collapsibility is NP-complete for $d \geq 4$**

A simplicial complex is *$d$-collapsible* if it can be reduced to an empty complex by repeatedly removing (collapsing) a face of dimension at most $d-1$ that is contained in a unique maximal face. We prove that the algorithmic question whether a given simplicial complex is $d$-collapsible is NP-complete for $d \geq 4$ and polynomial time solvable for $d \leq 2$.

As an intermediate step, we prove that $d$-collapsibility can be recognized by the greedy algorithm for $d \leq 2$, but the greedy algorithm does not work for $d \geq 3$.

A simplicial complex is *$d$-representable* if it is the nerve of a collection of convex sets in $\mathbb{R}^d$. The main motivation for studying $d$-collapsible complexes is that every $d$-representable complex is $d$-collapsible. We also observe that known results imply that analogical algorithmic question for $d$-representable complexes is NP-hard for $d \geq 2$.

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2009-937 J.-S. Sereni and M. Stehlík
**On the sextet polynomial of fullerenes**

We show that the sextet pattern count of every fullerene is strictly smaller than the Kekulé structure count. This proves a conjecture of Zhang and He [J. Math. Chem. 38(3):2005, p. 315–324].

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2009-938 H. Ardal, Z. Dvořák, V. Jungić, and T. Kaiser
**On a Rado Type Problem for Homogeneous Second Order Linear Recurrences**

In this note we introduce a Ramsey type function $S(r; a, b, c)$ as the maximum $s$ such that for any $r$-coloring of $\mathbb{N}$ there is a monochromatic sequence $x_1, x_2, \ldots, x_s$ satisfying a homogeneous second order linear recurrence $ax_i + bx_{i+1} + cx_{i+2} = 0$, $1 \leq i \leq s-2$. We investigate $S(2; a, b, c)$ and evaluate its values for a wide class of triples $(a, b, c)$.

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2009-939 Tomáš Valla (ed.)
**GRASTA 2009, Workshop on Graph Searching, Theory and Applications**

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2009-940 Jan Hubička, Jaroslav Nešetřil
**Homomorphism and embedding universal structures for restricted classes**

This paper unifies problems and results related to (embedding) universal and homomorphism universal structures. On the one side we give a new combinatorial proof of the existence of universal objects for homomorphism defined classes of structures (thus reproving a result of Cherlin, Shelah, Shi) and on the other side this leads to the new proof of the existence of dual objects (established by Nešetřil, Tardif). Our explicite approach has further applications to special structures such as variants of the rational Urysohn space. We also solve a related extremal problem which shows the optimality (of the used lifted arities) of our construction. Our method also relates to weakly indivisible homomorphism defined classes of structures.

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2009-941 C. Grosu and J. Hladký
**The extremal function for partial bipartite tilings**

For a fixed bipartite graph $H$ and given $\alpha \in (0,1)$ we determine the threshold $T_H(\alpha)$ which guarantees that any $n$-vertex graph with at least $T_H(\alpha)\binom{n}{2}$ edges contains $(1-o(1))\frac{\alpha}{v(H)}n$ vertex-disjoint copies of $H$.

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2009-942 M. DeVos, B. Mohar, and Robert Šámal
**Unexpected behaviour of crossing sequences**

The $n^{th}$ crossing number of a graph $G$, denoted $cr_n(G)$, is the minimum number of crossings in a drawing of $G$ on an orientable surface of genus $n$. We prove that for every $a > b > 0$, there exists a graph $G$ for which $cr_0(G) = a$, $cr_1(G) = b$, and $cr_2(G) = 0$. This provides support for a conjecture of Archdeacon et al. and resolves a problem of Salazar.

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2009-943 J. Foniok, K. Fukuda, and L. Klaus
**Combinatorial Characterizations of K-matrices**

We present a number of combinatorial characterizations of K-matrices. This extends a theorem of Fiedler and Pták on linear-algebraic characterizations of K-matrices to the setting of oriented matroids. Our proof is elementary and simplifies the original proof substantially by exploiting the duality of oriented matroids. As an application, we show that a simple principal pivot method applied to the linear complementarity problems with K-matrices converges very quickly, by a purely combinatorial argument.

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2009-944 T. Kaiser, A. King, and D. Král'
**Fractional total colourings of graphs of high girth**

Reed conjectured that for every $\epsilon > 0$ and $\Delta$ there exists $g$ such that the fractional total chromatic number of a graph with maximum degree $\Delta$ and girth at least $g$ is at most $\Delta + 1 + \epsilon$. We prove the conjecture for $\Delta = 3$ and for even $\Delta \geq 4$ in the following stronger form: For each of these values of $\Delta$, there exists $g$ such that the fractional total chromatic number of any graph with maximum degree $\Delta$ and girth at least $g$ is equal to $\Delta + 1$.

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2009-945 Robert Šámal
**Cubical coloring — fractional covering by cuts**

We introduce a new graph invariant that measures fractional covering of a graph by cuts. Besides being interesting in its own, it is useful for study of homomorphisms and tension-continuous mappings. We study the relations with chromatic number, bipartite density, and other graph parameters.

As a main result, we compute the parameter for infinitely many graphs based on hypercubes. These graphs play for our parameter the role that circular cliques play for the circular chromatic number. The fact, that the defined parameter attains on these graphs the 'correct' value suggests that the definition is a natural one.

In the proof we use the eigenvalue bound for maximum cut and a recent result of Engstrm, Frnqvist, Jonsson, and Thapper.

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2009-946 Aleš Pultr
**Why some fuzzyfications are easier then others**

When building a fuzzy variant of a theory of Gabriel-Ulmer type (such as a variety of algebras, a relational system, classical automata, etc.) it does not matter whether one models the theory directly in the universe of fuzzy sets or takes the corresponding crisp theory and fuzzifies it ex post.

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2009-947 J. Picado, and A. Pultr
**Cover quasi-uniformities in frames**

Quasi-uniformities (not necessarily symmetric uniformities) are usually studied via entourages (special neighbourhoods of the diagonal in $X \times X$) where one can simply forget about the symmetry requirement. This has been done successfully in the point-free context as well, but there is a demand for a covering approach, a.o. because the point-free representation of the square $X \times X$ is not without difficulties. Based on the (spatial) ideas from [T. E. Gantner and R. C. Steinlage, Characterizations of quasi-uniformities], a cover type quasi-uniformity was developped in [J. Frith, *Structured frames*] and other papers using biframes, the point-free variant of bitopologies. In this paper we show that this can be avoided and present a cover type quasi-uniformity structure enriching that of frame directly.

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2009-948 O. Zajíček, J. Sgall, and T. Ebenlendr
**Online Scheduling of Parallel Jobs on Hypercubes: Maximizing the Throughput**

We study the online problem of scheduling unit-time parallel jobs on hypercubes. A parallel job has to be scheduled between its release time and deadline on a subcube of processors. The objective is to maximize the number of early jobs. We provide a 1.6-competitive algorithm for the problem and prove that no deterministic algorithm is better than 1.4-competitive.

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2009-949 F. Kardoš, D. Král', and J.-S. Sereni
**The last fraction of a fractional conjecture**

Reed conjectured that for every $\varepsilon > 0$ and every integer $\Delta$, there exists $g$ such that the fractional total chromatic number of every graph with maximum degree $\Delta$ and girth at least $g$ is at most $\Delta + 1 + \varepsilon$. The conjecture was proven to be true when $\Delta = 3$ or $\Delta$ is even. We settle the conjecture by proving it for the remaining cases.

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2009-950 Milan Hladík
**On necessary efficient solutions in interval multiobjective linear programming**

We investigate multiobjective linear programming problems with objective coefficients varying inside given intervals. A feasible solution $x^*$ is called necessary efficient if it is efficient for all realizations of the interval coefficients. We show that the problem of testing necessary efficiency is NP-hard even in the case when $x^*$ is a non-degenerate basic solution. If we are given only one objective then the problem is polynomially solvable as long as $x^*$ is a non-degenerate basic solution, but it is NP-hard in the general case. Since the problem considered is computationally expensive, we propose one sufficient and also one necessary condition for necessary efficiency; this may significantly speed up the algorithms for testing necessary efficiency. We demonstrate usage of both conditions on illustrative examples.

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2009-951 P. Charbit and J.-S. Sereni
**The Fractional Chromatic Number of Zykov Products of Graphs**

Zykov designed one of the oldest known family of triangle-free graphs with arbitrary high chromatic number. We determine the fractional chromatic number of the Zykov product of a family of graphs. As a corollary, we deduce that the fractional chromatic numbers of the Zykov graphs satisfy the same recurrence relation as those of the Mycielski graphs, that is $a_{n+1} = a_n + \frac{1}{a_n}$. This solves a conjecture of Jacobs.

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2009-952 L. Esperet, F. Kardos, and D. Kral
**A superlinear bound on the number of perfect matchings in cubic bridgeless graphs**

Lovász and Plummer conjectured in the 1970's that cubic bridgeless graphs have exponentially many perfect matchings. This conjecture has been verified for bipartite graphs by Voorhoeve in 1979, and for planar graphs by Chudnovsky and Seymour in 2008, but in general only linear bounds are known. In this paper, we provide the first superlinear bound in the general case.

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2009-953 Z. Dvorak, D. Kral, and R. Thomas
**Deciding first-order properties for sparse graphs**

We present a linear time algorithm for deciding first-order logic (FOL) properties in classes of graphs with bounded expansion, which include proper minor-closed classes of graphs, and an almost linear time algorithm for deciding FOL properties in classes with locally bounded expansion, which include classes of graphs with locally bounded tree-width or locally excluding a minor. In addition, we design a fully dynamic data structure for testing the existence of short paths between prescribed vertices in such classes of graphs and a semidynamic data structure for finding subgraphs and, more generally, testing $\Sigma_1$-properties. All our results translate to corresponding classes of relational structures.

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2009-954 Martin Kupec (ed.)
**Abstracts of KAM-DIMATIA Series Year 2009**

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