GRASTA 2009

Workshop on Graph Searching, Theory and Applications
Valtice, October 5–9, 2009

Tomáš Valla (ed.)

Preface

The third Workshop on Graph Searching, Theory and Applications is held in hotel Hubertus located in the Valtice chateau in Czech Republic.

The goal of the event is to bring together various researchers and research groups working on problems related to Graph Searching, both from an applied and a theoretical point of view. Topics that are expected to be discussed at the workshop include

- Fugitive/Helicopter Search Games
- Monotonicity and Connectivity
- Marshal Games
- Conquest and Expansion Games
- Graph parameters related to graph searching
- New variants of graph searching
- Min-max theorems in graph searching
- Cops-Robbers games
- Pursuit-evasion games on graphs
- Applications of Graph searching in computer science

The workshop is sponsored by Center for Discrete Mathematics, Theoretical Computer Science and Applications (DIMATIA) and Institute for Theoretical Computer Science (ITI), both at Charles University in Prague.

We hope that the genius loci of the beautiful baroque chateau Valtice together with South-Moravian cultural tradition of spiritual power will purvey for the participants an inspiring working atmosphere yielding potential achievements in this interesting branch of Graph Theory.

Jan Kratochvíl and Geňa Hahn

Program

	Monday Oct 5 (arrival)
18:00	dinner
20:00	welcome party
	Tuesday Oct 6
10:00–12:00	D. Thilikos: Graph Searching Tutorial, Part I
12:00	lunch
14:00–16:00	D. Thilikos: Graph Searching Tutorial, Part II
18:00	dinner
	Wednesday Oct 7
9:00–10:00	P. Pralat: Chasing robbers on random graphs: The zigzag theorem
10:00–11:00	open problems
12:00	lunch
14:00–14:30	S. Ordyniak: Complexity and Monotonicity of Domination Search Games
15:00–17:00	discussions
17:00	progress report
18:00	dinner
	Thursday Oct 8
9:00–9:30	I. Adler: Games for width parameters and monotonicity
10:00–11:00	discussions
11:00	progress report
12:00	lunch
15:00	excursion to Lednice
19:00	banquet
	Friday Oct 9
9:00	F. Mazoit: Monotonicity in graph searching games
10:00–11:00	discussions
11:00	final report
12:00	lunch

List of invited talks

Dimitrios M. Thilikos

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Tutorial: An annotated bibliography on guaranteed graph searching

Graph searching encompasses a wide variety of combinatorial problems related to the problem of capturing a fugitive residing in a graph using the minimum number of searchers. In this annotated bibliography, we give an elementary classification of problems and results related to graph searching and provide a source of bibliographical references on this field.

Pawel Pralat

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Chasing robbers on random graphs: zigzag theorem

We study the vertex pursuit game of Cops and Robbers where cops try to capture a robber on the vertices of the graph. The minimum number of cops required to win on a given graph G is the cop number of G. We present asymptotic results for the game of Cops and Robber played on a random graph $G(n, p)$ for a wide range of $p = p(n)$. It has been shown that the cop number as a function of an average degree forms an intriguing zigzag shape.

Isolde Adler

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Games for width parameters and monotonicity

We introduce a search game for two players played on a “scenario” consisting of a ground set together with a collection of “feasible” partitions. This general setting allows us to obtain new characterisations of many width parameters such as rank-width, carving-width, matroid tree-width, and $GF(4)$-rank-width.

Monotonicity is a desirable property of search games that has attracted much interest. Hence a natural question arises: Can we characterise monotone search games? We are still far from resolving the question. Nevertheless, using ideas of Amini, Mazoit, Nisse and Thomasse, we exhibit a class of scenarios that allow monotonicity. This class includes the games for the parameters mentioned above.

Sebastian Ordyniak

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Complexity and Monotonicity of Domination Search Games

We study graph searching games where a number of cops try to capture a robber that is hiding in a system of tunnels modelled as a graph. While the current position of the robber is unknown to the cops, each cop can see a certain radius d around his position. For the case $d = 1$ these games have been studied by Fomin, Kratsch and Müller under the name domination games.

We are primarily interested in questions concerning the complexity and monotonicity of these games. We show that dominating games are computationally much harder than standard graph searching games where the cops only see their own vertex and establish strong non-monotonicity results for various notions of monotonicity which arise naturally in the context of domination games. Answering a question of given in the original paper about domination games, we show that there exists graphs for which the shortest winning strategy for a minimal number of cops must necessarily be of exponential length. On the positive side, we establish tractability results for graph classes of bounded degree.

Frédéric Mazoit

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Monotonicity in graph searching games

In graph searching games, a team of cops is trying to catch a fast robber in a graph. There are many variants of these games. For example, the robber may or may not be visible to the cops, the graph may be undirected or not... A wining strategy for the cops is monotone if the area available to the robber never increases, and a graph searching variant is monotone when the cops do not need extra agents to catch the robber with a monotone strategy. Several variants have been shown monotone but always with an ad-hoc proof. In this talk, I will present a recent framework in which one can prove monotonicity results, and related works.

Graph Searching Tutorial Lecture Notes

An annotated bibliography on guaranteed graph searching

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Abstract

Graph searching encompasses a wide variety of combinatorial problems related to the problem of capturing a fugitive residing in a graph using the minimum number of searchers. In this annotated bibliography, we give an elementary classification of problems and results related to graph searching and provide a source of bibliographical references on this field.

Introduction

Graph searching can be seen as a game between a fugitive and a set of searchers. The fugitive resides in a system of rooms and corridors represented by some graph G (vertices of G represent rooms while edges represent corridors between rooms). The purpose of searchers is to catch the fugitive who is trying to avoid capture. Depending on the dynamics of searchers and fugitives, phase restrictions, conditions of captures, global restrictions, visibility, etc. we obtain different models. The variants are either application driven motivated by problems in practice, or are inspired by foundational issues in Computer Science and Discrete Mathematics. Currently, the field on graphs searching is rapidly expanding and during the last years several new models, problems or approaches have appeared. In this annotated bibliography, we attempt to list and classify the main models and results in this field. Our classification is restricted to the classic line of *guaranteed* searching, initiated by Parsons and Petrov, where the searchers have to guarantee the capture without any probabilistic assumption on the game behavior.

Caves Mythology

The first mathematical formulation of graph searching is due to Torrence Parsons [126] and appeared in 1976. (See also [127].) The formulation was inspired by an earlier article of Breisch in Southwestern Cavers Journal [29] proposing a "speleotopological" approach for the problem of finding an explorer that is lost in a complicated system of dark caves. The question is to find the minimum number of searchers that can come out with a strategy of the guaranteed search, i.e. the strategy that allows them to rescue the explorer independent of his actions. In particular, the search strategy should be successful even if the explorer tries to avoid meeting with searchers. In this case we deal with a pursuit-evasion problem and this is why in the literature the searchers are called by pursuers or cops, and the explorer is called by evader, fugitive, or robber.

Parsons introduced the following pursuit-evasion problem: Let Γ be a straight line representation in R^3 of a finite connected graph G (i.e. topological graph embedded in R^3). A family $\Pi = \{x_i | 1 \leq i \leq k\}$ of continuous functions $x_i\colon [0,\infty) \to \Gamma$ is a search program for Γ if, for every continuous function $y\colon [0,\infty) \to \Gamma$ there exists a (smallest) $t(y) \in [0,\infty)$ and an $i \in \{1,2,\cdots,k\}$ such that $y(t(y)) = x_i(t(y))$. Here $y(t)$ may be thought of as the position at time t of an evader (explorer) in a cave (represented by Γ) while $x_i(t)$ is the position of the ith searcher, and $t(y)$ is the moment when the evader is caught. The minimum k that guarantees the existence of the search program on Γ was called the *search number of* Γ. The problem (and some results of Parsons) were later (independently) rediscovered in USSR by Nikolai Petrov [130] in slightly different settings. Petrov studied graph searching as a natural restriction of differential games in Euclidian spaces like the cossacks and the robber game [137] and the princess and the monster problem [99].

As it was proved by Golovach [73, 74], both problems of Parsons and Petrov are equivalent to the following discrete game played on graphs. Suppose that G is a graph representing a system of tunnels where an agile and omniscient fugitive with unbounded speed is hidden (alternatively, we can formulate the same problem considering that the tunnels are contaminated by some poisonous gas). Initially the whole graph is contaminated. The object of the game is to *clear* all edges, using one or more *searchers*. A *move* of a searcher can be one of the following: *placement* of the searcher on a vertex v, *removal* of the searcher from a vertex v and *sliding* of the searcher on a vertex v along an incident edge until its final placement to a neighbour vertex. An *search program* is a sequence of moves. In the edge search game, an edge $e = \{v,u\}$ of the graph is cleared if a searcher on v slides along $\{v,u\}$. A cleared edge become contaminated at some moment of searching (we say that the edge is *recontaminated*) if at that moment it can be connected by a path without searchers with a contaminated edge. (In terms of searching it means that the fugitive can run to already cleared area.) The object of the searchers in a search game is to clear all edges using an edge search program. The *edge search number* of a search program is the maximum number of searchers on the graph during any of its moves. The *edge search number*, $\mathbf{es}(G)$, of a graph G is the minimum edge search number over all the possible search programs on G.

A large number of the search variants studied so far where based on the above discrete model and where generated by restricting or enhancing the abilities of the searchers or of the fugitive. For surveys on graph searching, see [21, 63, 6]. Also, for a book on related problems, see [5].

Search variants

Node search and mixed search. The first variant of the edge search number was given by Kirousis and Papadimitriou in [104, 103] and is called *node search*. Here, the searchers cannot slide along edges and an edge is declared clean when both its endpoints are occupied by searchers. Another variant was given in Bienstock and Seymour in [23] and is called *mixed searching*. Here, sliding is permitted and and edge can be cleaned either by sliding or by simultaneous occupation of both its endpoints. This variant unifies the previous two as both edge search and node search can be easily reduced to mixed search. Also, as shown in [23] (see also [158]), all these three variants cannot differ by more than a small constant.

Visible vs lazy fugitives. In the original edge search game, as well as in node and mixed search, the fugitive is *invisible*, i.e. the searchers have no information on his current position. This means that the search strategy should be given in advance and does not take into account the possible responses of the fugitive after any of the searcher's moves. The case of a node searching of *visible* fugitive was studied by Seymour and Thomas in [149]. They call this a helicopter cop-and-robber game. The next variant of the classic setting considered the case where the invisible fugitive is lazy, in the sense that he moves only in the case where one of the searchers is planning to occupy the vertex where he currently is. This invisible-but-lazy variant was defined in [42] and was proved to be equivalent to the aforementioned visible-but-agile variant of Seymour and Thomas in [149]. A general variant, where the visibility is tuned by fixing the number of steps where the fugitive is visible, was called *non-deterministic search* [57]. Also, the extension of the classic game where multiple fugitives are considered was defined in [146]. This extension provided alternative way to tune between the visible and the invisible case.

Directed graphs. An active field of research on graph searching is to determine and study the analogues of all previous variants on directed graphs. Here, we have a variety of problems, depending not only on how searchers clear the edges (by sliding along edges or by placing on vertices) but also on how the searchers/fugitive traverse directed edges. Variants of edge and node searching on acyclic directed graphs where searchers may slide along an edge only in the indicated direction, but the fugitive can move in any direction, are studied by Nowakowski in [123]. Barát [16] studied the node search on directed graphs, where the fugitive is allowed to move only along directed paths and the searchers can be placed on any vertex. The edge-search and mixed-search variants of searching when searchers and the fugitive are allowed only to follow the directions are studied by Yang and Cao in [166]. Searching without jumping on directed graphs is done by Alspach et al. [7].

For the case when the fugitive is visible, there are also several variants of the game. Berwanger et al. [20] and Obdržálek [125] studied a variant of visible fugitive on directed graph with searchers jumping from vertex to vertex and fugitive allowed to move only along directed paths. Johnson et al. [100] studied a different variant where the fugitive is permitted to move only to vertices from which there exists a directed searcher-free path from his intended destination back to his current position. A variant with lazy fugitive was studied by Kreutzer and Hunter in [97]. Let us note that the equivalence between a lazy-but-invisible and agile-but-visible variants does not hold in directed graphs [97]. See also the thesis of Hunter for more discussions [98].

Hypergraphs. Graph searching on hypergraphs appeared as the Robber and Marshals game and was introduced by Gottlob, Leone and Scarcello in [86]. Here the robber is visible and moves on vertices just as in the classic search game, but searchers are now viewed as marshals that are able to place their men on hyperegdes of their choice. The Marshals are more powerful than classic searchers, as the cost of the search is now measured by the maximum number of hyperedges controlled by the marshals at each moment of the search. For a review on the applications of this game we refer to [85]. (See also [1, 2] and [88] for related results).

Connected search. Recall that the origin of graph searching considered the graph as a system of caves. Under this setting, the “cleaned” space should be connected. The edge searching variant where the cleaning of the graph happens in a connected way is defined in [18] (see also [17]). A natural question that appears is to estimate the “cost of connectivity”: the ratio between the number of searchers required in the connected case and the the number of searchers required without the connectivity requirement. Upper bounds on this ratio were given in the general case in [66] and for special graph classes in [18, 47, 49, 48, 65, 67, 120]. While there exist graphs where the price of connectivity tends asymptotically to be equal to 2, we dare to conjecture that, in general, the same value also constitutes a universal upper bound. A related problem on planar triangulations was studied in [124]. Connected searching with visible fugitive is discussed in [68].

Cop and robbers. A different, and somehow independent, branch of research on graph searching considered the case where the searchers (cops) and the fugitive (robber) move in rounds and when they move to a neighbor vertex. The minimum number of cops that guarantees cop's winning strategy on a graph G, is called the cop number of G. The part of Alspach's survey [6] is devoted to this parameter. See also the survey of Hahn [90]. The problem was defined by Winkler and Nowakowski [122], and Quilliot [143] who also characterized graphs where single cop has a winning strategy (see also [13, 32, 33]). A characterization of graphs with cop number k, $k > 1$, is an open problem. Aigner and Fromme proved that the cop number of a planar graph is at most 3 [4]. This result can be extended to graphs of bounded genus [144]. Schroeder [148] proved that for graphs of genus g, $\lfloor \frac{3g}{2} \rfloor + 3$ cops suffice to catch robber. It is not known if this bound is tight. Combinatorial (lower and upper) on the cop number results on different graph classes are discussed in [9, 10, 19, 30, 46, 69, 70, 91, 92, 93, 110, 117, 118, 145, 161]

Goldstein and Reingold [72] discussed the game on directed graphs. Different variations of the game with partial information can be found in [35, 37, 34, 39, 38, 43].

Alternative costs. In all above variants the target was to minimize the number of searchers that are present each moment on the graph. An alternative way to define the cost of a search strategy is to consider the sum of the searchers present on the graph over all steps of a search strategy. This was considered in [58]. Another way to evaluate the cost of a search strategy is to count the maximum number of steps a vertex is occupied by a searcher [61].

Games with restricted speed. A more realistic scenario to the original (non-discrete) definition of a graph searching game, where the maximum speeds of players are restricted, was defined by Petrov [131] who assumed that both pursuers P_i and the evader E possess simple motions in a topological graph Γ embedded in $\mathbf{R}^3$:

$$\begin{array}{lcr} (P_i) & : & \dot{x}_i = u_i,\, \|u_i\| \leq 1,\, i \in \{1, \ldots, n\}, \\ (E) & : & \dot{y} = u_0,\, \|u_0\| \leq \mu. \end{array}$$

($\|\cdots\|$ is the Euclidean norm.) Notice that the case of $\mu = +\infty$ (the evader can move arbitrarily fast) gives the original (non discrete) definition. Petrov [131], proved that when u_1 is a measurable function with an infinite number of discontinuities (jumps), for each Γ there is (sufficiently small) $\mu(\Gamma) > 0$ such that there is a search program of *one* searcher on Γ. Some upper bounds on $\mu(\Gamma) > 0$ sufficient for the existence of search program of one searcher are also given by Azamov & Norzhigitov in [15].

When admissible controls u_i and u_0 are piecewise constant functions defined on arbitrary segments $[0, T]$, the situation is different. For a given graph Γ and number μ, let $S_\mu(\Gamma)$ be the smallest number of searchers sufficient for the existence of a search program (with piecewise constant control functions) on Γ. The number $S_\mu(\Gamma)$ can be found only in exceptional cases. Searching on trees is discussed in [51, 52]. Computing of S_μ for some values of μ on graphs of Tetrahedron and Cube is discussed in [132, 109, 136, 84, 59]. For further results on the the numbers S_μ, see [50, 63, 53].

Finite speed was also considered in discrete games. In particular, Dendris et al. [42] studied the variant of a lazy invisible searching in the case where the fugitive, when moves, moves to positions of bounded distance.

Games with "radius of capture". Another generalization of edge searching is searching with radius of capture. In the game of the previous paragraph, let us put $\mu = +\infty$, i.e. we assume that the fugitive can move arbitrarily fast. Let ρ be some metric of Γ. Denote by ε a nonnegative number characterizing the "radius of capture". The evader E is *caught* by a pursuer P_i at a moment $t \in [0, T]$, if $\rho(x_i(t), y(t)) \leq \varepsilon$. A family of trajectories of n searchers $x_i\colon [0, T] \to \Gamma$, $i \in \{1, 2, \ldots, n\}$ is called a *winning* program if for every trajectory of the fugitive $y\colon [0, T] \to \Gamma$, there exist $t \in [0, T]$ and $i \in \{1, 2, \ldots, n\}$, such that $\rho(x_i(t), y(t)) \leq \varepsilon$. Thus the searchers have "visibility" ε and they catch the fugitive when they see him. The problem is to determine the minimal number of pursuers that have a winning program and let us denote this number by $S^\varepsilon(\Gamma)$. Again, for $\varepsilon = 0$ the number $S^\varepsilon(\Gamma)$ is the search number of Γ as it is defined by Parsons and Petrov.

The study of $S^\varepsilon(\Gamma)$ when the metric ρ on Γ is induced by the Euclidian space was initiated by Golovach [75, 74]. For example, it was shown that if all edges of a topological graph Γ are of unit length, then for $\varepsilon < 1/4$, $S^\varepsilon(\Gamma) = \mathbf{es}(\Gamma)$. The study of $S^\varepsilon(\Gamma)$ on topological graphs with edges of unit length consisting of all edges and vertices of Tetrahedron, Cube and Octahedron respectively is performed in [59, 133]. Searching with a radius of capture can be naturally generalized to searching of polygons. See [89, 107, 139, 163, 155, 159, 164, 40, 156, 154, 171] for some geometric search problems. The case when ρ is a shortest path metric of the graph were studied in [141, 142] for $\varepsilon = 1$ and for $\varepsilon \geq 1$ in [83, 82, 134].

Another (discrete) variant variant of this game, when searchers do not slide but only jump and $\epsilon = 1$, is called the *domination search game*. This variant is studied in [62, 3]. A variant of searching on grids (or systems of streets) where searchers can see a fugitive if they are on the same street was studied in [41, 119, 153].

Searching with counteraction. Searching with counteraction is a variant of Cop and Robber game with invisible Robber. However the difference is that Robber can kill Cops by placing himself at his move on a vertex occupied by a Cop. When a cop is killed (removed from the playing board), by his removal the cop player retrieve information on robber's position at this move. Such type of games were studied in [12, 11, 140, 172, 135, 101].

Miscellaneous variations. Single step edge-search game when searchers are allowed to make only one step is studied in [31, 94, 96, 95]. The relations between the number of searchers and number of steps in search program are discussed in [28]. Clarke and Nowakowski looked at searching with radar, i.e. some places in a graph where fugitive is seen [36]. Searching with distributed computing settings is studied in [24, 121]. Different search problems (with restricted speed) related to eavesdropping games are in [71]. The problem with one searcher flying from vertex to vertex on a helicopter is discussed in [54, 55, 56]. A version of searching where recontamination of an unguarded vertex occurs only if a majority of its neighbors is contaminated, is studied in [108].

Monotonicity

A desirable characteristic of a search strategy is to be never forced to search again an already searched area. In other words, to prevent the robbers to enter again the area from where they had been expelled. This property is called *monotonicity*. Not all graph searching variants are monotone. To distinguish which are and which are not appears to be a hard question. Actually, some of the most complicated results in graph searching were devoted to monotonicity and all of them where directly linked to the Graph Minors project (for a survey, see [147]).

Clearly, the monotonicity property can gives us information on the complexity of a graph searching problem as it provides a polynomial space certificate for proving membership in NP (we just consider only monotone strategies). The first monotonicity result concerned edge searching and was proved by LaPaugh and appeared in [106] (the result was first announced in [105]). The next step (based on the result of LaPaugh) was given by Kirousis and Papadimitriou, who proved in [104] the monotonocity of node searching. A break-through in proving monotonicity came in [23] where Bienstock and Seymour proved the monotonicity of mixed searching using a simpler technique than the one in [106] (see also [22]). This result also implied the monotonicity of the previous two classic variants as, according to [23], both edge and node searching can be easily reduced to the mixed searching.

The next step was to prove the monotonicity in the case of a visible fugitive. This proof was given by Seymour and Thomas in [149] and was based on the introduction of screens (also known as brambles) as certificates of escaping strategies. The analogue of these structures for the invisible fugitive case was introduced in [22] (called blockages) and generalized to a general nonotonicity criterion in [64]. Recently, a monotonicity proof unifying the cases where the fugitive is visible or invisible was proposed by Mazoit and Nisse in [113].

On directed graphs different modifications of searching with invisible fugitive are monotone [16, 166, 167, 168].

Certainly, there are cases where monotonicity does not hold. This was proved to be the case for connected search in [169] and [68]. Also, as observed in [146], the same holds when many visible robbers are considered. Finally, it also follows that the Marshals game is not monotone, as proved in [1].

Parameters, complexity and algorithms

The first complexity and algorithmic results on the edge search number are due to Megiddo et al. [114], who proved that it is NP-complete in general and solvable in linear time on trees (however, some key algorithmic ideas were already present in the original papers of Parsons [127, 126]). The NP-completeness of node search is obtained by a reduction from edge search provided in [104]. Since both edge and node search can be reduced to mixed search, it follows that mixed search is also NP-complete [23]. For a fixed k, it can be decided in linear time if the edge (node and mixed) search number of a graph is at most k [27, 160]. Similar results (by equivalence to treewidth) are known for searching of visible and lazy fugitives [25].

A common practice for examining the complexity of a search variant is to prove its monotonicity and then find (or define) a graph parameter that is equivalent to its monotone version. Then the problem is reduced to the study of the parameter. The first such parameter was pathwidth that, combining the results in [103, 104, 102, 115], appears to be equal to the node search number minus one (thus, the result on the NP-completeness of pathwidth in [14] gives an alternative way to prove the NP-completeness of node search and mixed search). Another version of searching, namely, strong mixed searching, equivalent to pathwidth, is given by Yang [165]. Next, in [158], Takahashi, Ueno, and Kajitani introduced the proper-pathwidth as a parameter equal to the mixed search number.

The case of a visible and agile fugitive was shown to be equal to the treewidth minus one. Thus, the NP-completeness of this variant follows from its monotonicity and the NP-hardness of treewidth [14]. As proved in [42], the same parameter, is equivalent to the lazy-but-invisible fugitive while the special case of a lazy fugitive of unit speed is equivalent to the degeneracy of a graph (that can be computed in polynomial time). Also, a decomposition based analogue of non-deterministic search was given in [57].

As shown in [86], the monotone version of the Marshals game is equivalent to the parameter of hypertree-width. Grohe and Marx defined a fractional version of hypertree-width, which is related to a fractional Marshals game [88]. The complexity of variations of the Marshals game is discussed in [87].

Let us also mention other graph parameters that can be expressed or are related to some versions of graph searching. Cutwidth is related to edge-searching (on graphs of vertex degree at most 3 these parameters are equal) [111]. Relations between topological bandwidth and node search numbers can be found in [112]. Profile of a graph is equal to the cost of the node-search [58]. Bandwidth and proper pathwidth can be defined via helicopter search problems [54, 56]. A game theoretic approach to the tree-span parameter is given in [61]. For directed graphs, the search interpretation of directed pathwidth, directed treewidth, Kelly-width, DAG-width can be found in [16, 20, 125, 100, 97].

Branchwidth of a planar graph can be expressed via ratcatcher game [150]. The survey of Bodlaender [26] has a part on relations between graph searching and different "width" graph parameters.

To mention all algorithmic and combinatorial results for all above parameters is out of scope of this bibliography. See [8, 44, 45, 67, 66, 57, 60, 151, 158, 160, 162] for results that were directly linked with search parameters. For results on graph searching on special graph classes, see [60, 77, 76, 80, 81, 116, 128, 129, 157, 170]. Minimal (with respect to minors or subgraphs) graphs with small search numbers are characterized in [114, 138]. The problems of enumerating all minimal trees (for different search variants) are solved in [78, 79, 152, 160]. For obstructions or obstruction characterisations of graphs with small search numbers see [150, 160, 149, 64, 22].

The computational complexity of Cop and Robber game is an open question. Determining whether the cop number of a graph on n vertices is at most k can be done by a backtracking algorithm which is polynomial for fixed k [19, 92]. Goldstein and Reingold [72] proved that the version of the game on directed graphs is EXPTIME-complete. Also they have shown that the version of the game on undirected graphs when cops and robbers are given their initial positions is also EXPTIME-complete.

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