Abstracts of KAM-DIMATIA Series Year 2008

Tomáš Valla (ed.)

2008-844 D. Král', O. Pangrác, J. Sereni, and R. Škrekovski
Long cycles in fullerene graphs

It is conjectured that every fullerene graph is hamiltonian. Jendrol' and Owens proved [J. Math. Chem. 18 (1995), pp. 83–90] that every fullerene graph on n vertices has a cycle of length at least $4n/5$. In this paper we, improve this bound to $5n/6 - 2/3$.

2008-845 B. Lidický
On 3-choosability of plane graphs without 6-, 7- and 8-cycles

A graph is k-choosable if it can be colored whenever every vertex has a list of available colors of size at least k. It is a generalization of graph coloring where all vertices do not have same available colors. We show that every triangle-free plane graph without 6-, 7-, and 8-cycles is 3-choosable.

2008-846 D. Král', P. Nejedlý, and R. Šámal
Short Cycle Covers of Cubic Graphs

The Shortest Cycle Cover Conjecture asserts that the edges of every bridgeless graph with m edges can be covered by cycles of total length at most $7m/5 = 1.4m$. We show that every cubic bridgeless graph has a cycle cover of total length at most $34m/21 \approx 1.619m$.

2008-847 T. Kaiser, D. Král', B. Lidický, and P. Nejedlý
Short Cycle Covers of Graphs with Minimum Degree Three

The Shortest Cycle Cover Conjecture asserts that the edges of every bridgeless graph with m edges can be covered by cycles of total length at most $7m/5 = 1.4m$. We show that every bridgeless graph with minimum degree three that contains m edges has a cycle cover comprised of three cycles of total length at most $44m/27 \approx 1.6296m$; this extends a bound of Fan [J. Graph Theory 18 (1994), 131–141] for cubic graphs to the class of all graphs with minimum degree three.

2008-848 J. Fiala and P. Golovach
Complexity of the Packing Coloring Problem of Trees

Packing coloring is a partitioning of the vertex set of a graph with the property that vertices in the i-th class have pairwise distance greater than i. We solve an open problem of Goddard et al. and show that the decision whether a tree allows a packing coloring with at most k classes is NP-complete.

We accompany this negative result by a polynomial time algorithm for trees for closely related variant of the packing coloring problem where the lower bounds on the distances between vertices inside color classes are determined by an infinite nondecreasing sequence of bounded integers.

2008-849 J. Sereni
Equitable colourings of graphs with bounded density and given girth

We study the equitable chromatic number of graphs of minimum degree at least 2, lower-bounded girth and upper-bounded density. As direct corollaries, we obtain that the equitable chromatic number of a planar graph with minimum degree at least 2 is at most 3 if the graph has girth at least 14, and at most 4 if the graph has girth at least 10. This improves the previously known bounds, obtained by Wu and Wang [Discrete Mathematics, 308(5-6):985–990, 2008].

2008-850 J. Foniok and J. Nešetřil
Splitting finite antichains in the homomorphism order

A structural condition is given for finite maximal antichains in the homomorphism order of relational structures to have the splitting property. It turns out that non-splitting antichains appear only at the bottom of the order. Moreover, we examine looseness and finite antichain extension property for some subclasses of the homomorphism poset. Finally, we take a look at cut-points in this order.

2008-851 J. Nešetřil and Y. Nigussie

Finite dualities and map-critical graphs on a fixed surface

Let $\mathcal{K}$ be a class of graphs. Then, $\mathcal{K}$ is said to have a finite duality if there exists a pair $(\mathcal{F}, U)$, where $U \in \mathcal{K}$ and $\mathcal{F}$ is a finite set of graphs, such that for any graph G in $\mathcal{K}$ we have $G \leq U$ if and only if $F \not\leq G$ for all $F \in \mathcal{F}$ (" $\leq$ " is the homomorphism order). We prove that the class of planar graphs has no finite duality except for two trivial cases. We also prove that a 5-colorable toroidal graph U obtains a finite duality on a given fixed surface if and only if the core of U is K_5. In a sharp contrast, for a higher genus orientble surface S we show that Thomassen's result [Color-Critical Graphs on a Fixed Surface, J. Combin. Theory, Series A, Vol. 70(1997), 67-100] implies that the class, $\mathcal{G}(S)$, of all graphs embeddable in S has a number of finite dualities. Equivalently, our first result shows that for every planar core graph H (except K_1 and K_4) there are infinitely many minimal planar obstructions for H-coloring, whereas our later result gives a converse of Thomassen's theorem for 5-colorable graphs on the torus.

2008-852 Z. Dvořák, T. Kaiser, D. Král', and J. Sereni

A note on antisymmetric flows in graphs

We prove that any orientation of a graph without bridges and directed 2-edge-cuts admits a $\mathbb{Z}_2^3 \times \mathbb{Z}_3^9$-antisymmetric flow, which improves the bounds obtained by DeVos, Johnson and Seymour, and DeVos, Nešetřil and Raspaud.

2008-853 D. Král', O. Serra, and L. Vena

A combinatorial proof of the Removal Lemma for Groups

Green [Geometric and Functional Analysis 15 (2005), 340–376] established a version of the Szemerédi Regularity Lemma for abelian groups and derived the Removal Lemma for abelian groups as its corollary. We provide another proof of his Removal Lemma that allows us to extend its statement to all finite groups. We also discuss possible extensions of the Removal Lemma to systems of equations.

2008-854 D. Král', P. Nejedlý, and X. Zhu

Choosability of Squares of K_4-minor Free Graphs

Lih, Wang and Zhu [Discrete Math. 269 (2003), 303–309] proved that the chromatic number of the square of a K_4-minor free graph with maximum degree Δ is bounded by $\lfloor 3\Delta/2 \rfloor + 1$ if $\Delta \geq 4$, and is at most $\Delta + 3$ for $\Delta \in \{2, 3\}$. We show that the same bounds hold for the list chromatic number of squares of K_4-minor free graphs. The same result was also proved independently by Hetherington and Woodall.

2008-855 J. Matoušek, A. Přívětivý, and P. Škovroň
How many points can be reconstructed from k projections?

Let A be an n-point set in the plane. A *discrete X-ray* of A in direction u gives the number of points of A on each line parallel to u. We define $F(k)$ as the maximum number n such that there exist k directions $u_1, \ldots, u_k$ such that every set of at most n points in the plane can be uniquely reconstructed from its discrete X-rays in these directions. A simple "cube" construction shows $F(k) \leq 2^{k-1}$. We establish the lower bound $F(k) \geq 2^{\Omega(k/\log k)}$ by reducing the problem through linear algebra to a graph-theoretic question, for which we then obtain an almost tight bound. As a part of the proof we establish a result in extremal theory that allows one to conclude that, under certain conditions, a graph has only at most a logarithmic density, which may be of independent interest. We also improve the upper bound to $F(k) \leq O(1.81712^k)$ (or $O(1.79964^k)$ if we allow A to be a multiset).

2008-856 J. Matoušek and M. Tancer
On the gap between representability and collapsibility

A simplicial complex K is called *d-representable* if it is the nerve of a collection of convex sets in R^d; K is *d-collapsible* if it can be reduced to an empty complex by repeatedly removing a face of dimension at most $d-1$ that is contained in a unique maximal face; and K is *d-Leray* if every induced subcomplex of K has vanishing homology of dimension d and larger.

It is known that d-representable implies d-collapsible implies d-Leray, and no two of these notions coincide for $d \geq 2$. The famous Helly theorem and other important results in discrete geometry can be regarded as results about d-representable complexes, and in many of these results "d-representable" in the assumption can be replaced by "d-collapsible" or even "d-Leray".

We investigate "dimension gaps" among these notions, and we construct, for all $d \geq 1$, a $2d$-Leray complex that is not $(3d-1)$-collapsible and a d-collapsible complex that is not $(2d-2)$-representable. In the proofs we obtain two results of independent interest: (i) The nerve of every finite family of sets, each of size at most d, is d-collapsible. (ii) If the nerve of a simplicial complex K is d-representable, then K embeds in R^d.

2008-857 V. Franěk and J. Matoušek

Computing D-convex hulls in the plane

A function $f\colon R^d \to R$ is called *D-convex*, where D is a set of vectors in R^d, if its restriction to each line parallel to a nonzero $v \in D$ is convex. The *D-convex hull* of a compact set $A \subset R^d$ is the intersection of the zero sets of all nonnegative D-convex functions that are 0 on A. Matoušek and Plecháč provided an algorithm for computing the D-convex hull of a finite set in R^d for D consisting of d linearly independent vectors (in this case one speaks about *separately convex* hulls). Here we present a (polynomial-time) algorithm for the D-convex hull of a finite point set in the plane for arbitrary finite D.

2008-858 J. Matoušek

Removing degeneracy in LP-type problems revisited

LP-type problems is a successful axiomatic framework for optimization problems capturing, e.g., linear programming and the smallest enclosing ball of a point set. In [Matoušek and Škovroň, *Theory of Computing* 3(2007) 159–177] it was proved that in order to remove degeneracies of an LP-type problem, we sometimes have to increase its combinatorial dimension by a multiplicative factor of at least $1+\varepsilon$ for a certain small positive constant ε. The proof went by checking the unsolvability of a system of linear inequalities, with several pages of calculations.

Here we prove by a short topological argument that the dimension sometimes has to increase at least twice. We also construct 2-dimensional LP-type problems with $-\infty$ for which removing degeneracies forces arbitrarily large dimension increase.

2008-859 J. Matoušek

LC reductions yield isomorphic simplicial complexes

We say that a vertex v of a finite simplicial complex K is *LC-removable* if the link of v is a cone, and that K is *LC-irreducible* if it has no LC-removable vertices. Answering a question of Civan and Yalçın [*J. Comb.*

Theory Ser. A(2007), doi:10.1016/j.jcta.2007.02.001], we prove that all LC-irreducible simplicial complexes that can be obtained from a given K by repeatedly deleting LC-removable vertices (plus all simplices containing them) are isomorphic.

2008-860 J. Matoušek
On variants of the Johnson–Lindenstrauss lemma

The Johnson–Lindenstrauss lemma asserts that an n-point set in any Euclidean space can be mapped to a Euclidean space of dimension $k = O(\varepsilon^{-2} \log n)$ so that all distances are preserved up to a multiplicative factor between $1 - \varepsilon$ and $1 + \varepsilon$. Known proofs obtain such a mapping as a linear map $R^n \to R^k$ with a suitable random matrix. We give a simple and self-contained proof of a version of the Johnson–Lindenstrauss lemma that subsumes a basic versions by Indyk and Motwani and a version more suitable for efficient computations due to Achlioptas. (Another proof of this result, slightly different but in a similar spirit, was given independently by Indyk and Naor.) An even more general result was established by Klartag and Mendelson using considerably heavier machinery.

Recently Ailon and Chazelle showed, roughly speaking, that a good mapping can also be obtained by composing a suitable Fourier transform with a linear mapping that has a *sparse* random matrix M; a mapping of this form can be evaluated very fast. In their result the nonzero entries of M are normally distributed. We show that the nonzero entries can be chosen as random ± 1, which further speeds up the computation. We also discuss the case of embeddings into R^k with the ℓ_1 norm.

2008-861 J. Cibulka, J. Hladký, M.A. LaCroix, and D.G. Wagner
A combinatorial proof of Rayleigh monotonicity for graphs

We give an elementary, self-contained, and purely combinatorial proof of the Rayleigh monotonicity property of graphs.

2008-862 F. Havet, D. Král', J. Sereni, and R. Škrekovski
Facial colorings using Hall's Theorem

A vertex coloring of a plane graph is ℓ-facial if every two distinct vertices joined by a facial walk of length at most ℓ receive distinct colors. It has been conjectured that every plane graph has an ℓ-facial coloring with at most $3\ell + 1$ colors. We improve the currently best known bound and show that every plane graph has an ℓ-facial coloring with at most $\lfloor 7\ell/2 \rfloor + 6$ colors. Our proof uses the standard discharging technique, however, in the reduction part we have successfully applied Hall's Theorem, which seems to be quite an innovative approach in this area.

2008-863 J. Nešetřil, P. Ossona de Mendez
Structural Properties of Sparse Graphs

2008-864 D. Zeps
Combinatorial map as multiplication of combinatorial knots

We show that geometrical map can be expressed as multiplication of combinatorial maps, i.e. map P is equal to multiplication of its knot, inner knot's square and trivial knot ($= \mu \cdot \nu^2 \cdot \pi_1$).

2008-865 J. Nešetřil and P. Ossona de Mendez
First Order Properties on Nowhere Dense Structures

A set A of vertices of a graph G is called d-scattered in G if no two d-neighborhoods of (distinct) vertices of A intersect. In other words, A is d-scattered if no two distinct vertices of A have distance at most $2d$. This notion was isolated in the context of finite model theory by Gurevich and recently it played a prominent role in the study of homomorphism preservation theorems for special classes of structures (such as minor closed families). This in turn led to the notions of wide, almost wide and quasi-wide classes of graphs. It has been proved previously that minor closed classes and classes of graphs with locally forbidden minors are examples of such classes and thus (relativized) homomorphism preservation theorem holds for them. In this paper we show that (more general) classes with bounded expansion and (newly defined) classes with bounded local expansion and even (very general) nowhere dense classes are quasi wide. This not only strictly generalizes the previous results but it also provides new proofs and algorithms for some of the old results. It appears that bounded expansion and nowhere dense classes are perhaps a proper setting for investigation of wide-type classes as in several instances we obtain a structural characterization. This also puts classes of bounded expansion in the new context. Our motivation stems from finite dualities. As a corollary we obtain that any homomorphism closed first order definable property restricted to a bounded expansion class is a restricted duality.

2008-866 D. Král' and D. B. West
Chromatic number for a generalization of Cartesian product graphs

Let $\mathcal{G}$ be a class of graphs. The *d-fold grid over* $\mathcal{G}$, denoted $\mathcal{G}^d$, is the family of graphs obtained from d-dimensional rectangular grids of vertices by placing a graph from $\mathcal{G}$ on each of the lines parallel to one of the axes. Thus each vertex belongs to d of these subgraphs. Let $f(\mathcal{G}; d) = \max_{G \in \mathcal{G}^d} \chi(G)$. If each graph in $\mathcal{G}$ is k-colorable, then $f(\mathcal{G}; d) \leq k^d$. We show that this bound is best possible by proving that $f(\mathcal{G}; d) = k^d$ when $\mathcal{G}$ is the class of all k-colorable graphs. We also show that $f(\mathcal{G}; d) \geq \left\lfloor \sqrt{\frac{d}{6 \log d}} \right\rfloor$ when $\mathcal{G}$ is the class of graphs with at most one edge, and $f(\mathcal{G}; d) \geq \left\lfloor \frac{d}{6 \log d} \right\rfloor$ when $\mathcal{G}$ is the class of graphs with maximum degree 1.

2008-867 D. Král', J. Sereni, and M. Stiebitz
A new lower bound on the number of perfect matchings in cubic graphs

We prove that every n-vertex cubic bridgeless graph has at least $n/2$ perfect matchings and give a list of all 17 such graphs that have less than $n/2 + 2$ perfect matchings.

2008-868 J. Foniok and C. Tardif
Adjoint functors and tree duality

A family $\mathcal{T}$ of digraphs is a *complete set of obstructions* for a digraph H if for an arbitrary digraph G the existence of a homomorphism from G to H is equivalent to the non-existence of a homomorphism from any member of $\mathcal{T}$ to G. A digraph H is said to have *tree duality* if there exists a complete set of obstructions $\mathcal{T}$ consisting of orientations of trees. We show that if H has tree duality, then its arc graph δH also has tree duality, and we derive a family of tree obstructions for δH from the obstructions for H.

Furthermore we generalise our result to right adjoint functors on categories of relational structures. We show that these functors always preserve tree duality, as well as polynomial CSPs and the existence of near-unanimity functions.

2008-869 T. Marshall
Homomorphism bounds for oriented planar graphs II

We show that, if H is an oriented graph which admits a homomorphism from each oriented planar graph, then the order of H is at least 18; thus there exists an oriented planar graph with oriented chromatic number at least 18.

2008-870 R. Ball, A. Pultr, and J. Sichler
Tame parts of free summands in coproducts of Priestley spaces

It is well known that a sum (coproduct) of a family $\{X_i : i \in I\}$ of Priestley spaces is a compactification of their disjoint union, and that this compactification in turn can be organized into a union of pairwise disjoint order independent closed subspaces X_u, indexed by the ultrafilters u on the index set I. The nature of those subspaces X_u indexed by the free ultrafilters u is not yet fully understood.

In this article we study a certain dense subset $X_u^\partial \subseteq X_u$ satisfying exactly those sentences in the first-order theory of partial orders which are satisfied by almost all of the X_i's. As an application we present a complete analysis of the coproduct of an increasing family of finite chains, in a sense the first non-trivial case which is not a Čech-Stone compactification of the disjoint union $\bigcup_I X_i$. In this case, all the X_u's with u free turn out to be isomorphic under the Continuum Hypothesis.

2008-871 B. Banaschewski and A. Pultr
Epimorphisms of metric frames

This paper deals with several aspects of epimorphisms in the category **MFrm** of metric frames and contractive homomorphisms. In particular, it is shown that

(i) the epicomplete metric frames are uniquely determined by
the power-set lattices of sets,
(ii) episurjective is the same as Boolean,
(iii) a metric frame has an epicompletion iff it is spatial, and
(iv) the subcategory of epicomplete L in **MFrm** is reflective.

Moreover, we show that the counterpart of the latter does not hold for uniform frames.

2008-872 S. Felsner and M. Pergel
The Complexity of Sorting with Networks of Stacks and Queues

We consider a sorting problem on networks whose nodes are storage elements of type stack or queue. A railway switchyard could be an instance of such a network. Given is an input node where a permutation of items 1 to n is delivered and an output node where they are expected in sorted order. How many moves, where an item is transfered from one node to an adjacent node, are needed in the worst case for the sorting? Among others we have the following results: A characterization of networks where the sorting complexity is $\Theta(n \log n)$. A lower bound of $\Omega(n^{2-\varepsilon})$ for the network consisting of only two stacks that can exchange items.

2008-873 J. Nešetřil
Many facets of dualities

In this paper we survey results related to homomorphism dualities for graphs, and more generally, for finite structures. This is related to some of the classical combinatorial problems, such as colorings of graphs and hypergraphs, and also to recently intensively studied Constraint Satisfaction Problems. On the other side dualities are related to the descriptive complexity and First Order definability as well as to universal graphs. And in yet another context they can be expressed as properties of the homomorphism order of structures. In the contemporary context homomorphism dualities are a complex area and it is our aim to describe some of the main ideas only. However we introduce the four conceptually different proofs of the existence of duals thus indicating the versatility of this notion. Particularly we describe setting of restricted dualities and the role of bounded expansion classes.

2008-874 J. Hladký and D. Piguet
Loebl-Komlós-Sós Conjecture: dense case

We prove a version of the Loebl-Komlós-Sós Conjecture for dense graphs. For any $q > 0$ there exists a number $n_0 \in \mathbb{N}$ such that for any $n > n_0$ and $k > qn$ the following holds: if G be a graph of order n with at least $n/2$ vertices of degree at least k, then any tree of order $k+1$ is a subgraph of G.

2008-875 F. Havet, M. Klazar, J. Kratochvíl, D. Kratsch, and M. Liedloff
Exact algorithms for $L(2,1)$-labeling of graphs

The notion of distance constrained graph labelings, motivated by the Frequency Assignment Problem, reads as follows: A mapping from the vertex set of a graph $G = (V, E)$ into an interval of integers $\{0, \ldots, k\}$ is an $L(2,1)$-labeling of G of span k if any two adjacent vertices are mapped onto integers that are at least 2 apart, and every two vertices with a common neighbor are mapped onto distinct integers. It is known that for any fixed $k \geq 4$, deciding the existence of such a labeling is an NP-complete problem. We present exact exponential time algorithms that are faster than the naive $O((k+1)^n)$ algorithm that would try all possible mappings. The improvement is best seen in the first NP-complete case of $k = 4$ – here the running time of our algorithm is $O(1.3006^n)$. Furthermore we show that dynamic programming can be used to establish an $O(3.8730^n)$ algorithm to compute an optimal $L(2,1)$-labeling.

2008-876 P. Gregor and R. Škrekovski
Long cycles in hypercubes with distant faulty vertices

In this paper, we study long cycles in induced subgraphs of hypercubes obtained by removing a given set of faulty vertices such that every two faults are distant. First, we show that every induced subgraph of Q_n with minimum degree $n-1$ contains a cycle of length at least $2^n - 2f$ where f is the number of removed vertices. This length is the best possible when all removed vertices are from the same bipartite class of Q_n. Next, we prove that every induced subgraph of Q_n obtained by removing vertices of some given set M of edges of Q_n contains a Hamiltonian cycle if every two edges of M are at distance at least 3. The last result shows that the shell of every linear code with odd minimum distance at least 3 contains a Hamiltonian cycle. In all these results we obtain significantly more tolerable faulty vertices than in the previously known results. We also conjecture that every induced subgraph of Q_n obtained by removing a balanced set of vertices with minimum distance at least 3 contains a Hamiltonian cycle.

2008-877 D. Dimitrov, T. Dvořák, P. Gregor, and R. Škrekovski
Gray Codes Faulting Matchings

A (cyclic) n-bit Gray code is a (cyclic) ordering of all 2^n binary strings of length n such that consecutive strings differ in a single bit. Equivalently, an

n-bit Gray code can be viewed as a Hamiltonian path of the n-dimensional hypercube Q_n, and a cyclic Gray code as a Hamiltonian cycle of Q_n. In this paper we study Hamiltonian paths and cycles of Q_n avoiding a given set of faulty edges that form a matching, briefly called (cyclic) Gray codes faulting a given matching. Given a matching M and two vertices u, v of Q_n, $n \geq 4$, our main result provides a necessary and sufficient condition, expressed in terms of forbidden configurations for M, for the existence of a Gray code between u and v faulting M. As a corollary, we obtain a similar characterization for a cyclic Gray code faulting M. In particular, in case that M is a perfect matching, Q_n has a (cyclic) Gray code faulting M if and only if $Q_n - M$ is a connected graph. This complements a recent result of Fink, who proved that every perfect matching of Q_n can be extended to a Hamiltonian cycle.

2008-878 J. Miškuf, R. Škrekovski, and M. Tancer
Backbone colorings of graphs with bounded degree

We study backbone colorings, a variation on classical vertex colorings: Given a graph G and a spanning subgraph H of G (the backbone of G), a backbone coloring for G and H is a proper vertex k-coloring of G in which the colors assigned to adjacent vertices in H differ by at least 2. The minimal $k \in \mathbb{N}$ for which such a coloring exists is called the backbone chromatic number of G. We show that for a graph G of maximum degree Δ with the backbone graph being a d-degenerated subgraph of G, the backbone chromatic number is at most $\Delta + d + 1$ and moreover, in the case when the backbone graph being a matching we prove that backbone chromatic number is at most $\Delta + 1$. We also present examples where these bounds are attained.

Finally, the asymptotic behavior of the backbone chromatic number is studied regarding the degrees of G and H. We prove for any sparse graph G that if the maximum degree of a backbone graph is small compared to the maximum degree of G, then the backbone chromatic number is at most $\Delta(G) - \sqrt{\Delta(G)}$.

2008-879 J. Miškuf, R. Škrekovski, and M. Tancer
Backbone Colorings and Generalized Mycielski's Graphs

For a graph G and its spanning tree T the *backbone chromatic number*, $\mathrm{BBC}(G, T)$, is defined as the minimum k such that there exists a coloring

$c\colon V(G) \to \{1, 2, \ldots, k\}$ satisfying $|c(u) - c(v)| \geq 1$ if $uv \in E(G)$ and $|c(u) - c(v)| \geq 2$ if $uv \in E(T)$.

Broersma et al. [Backbone colorings for graphs: Tree and path backbones. *J. Graph Theory*, 55(2):137–152, 2007] asked whether there exists a constant c such that for every triangle-free graph G with an arbitrary spanning tree T the inequality $\mathrm{BBC}(G, T) \leq \chi(G) + c$ holds. We answer this question negatively by showing the existence of triangle-free graphs R_n and their spanning trees T_n such that $\mathrm{BBC}(R_n, T_n) = 2\chi(R_n) - 1 = 2n - 1$.

In order to answer the question we obtain a result of independent interest. We modify the well known Mycielski's construction and construct triangle-free graphs J_n, for every integer n, with chromatic number n and 2-tuple chromatic number $2n$ (here 2 can be replaced by any integer t).

2008-880 Y. Nigussie

Structural description for Kruskal-Friedman ideals of finite trees

In [Structural descriptions of lower ideals of trees, Contemporary Mathematics, 147 (1993), 525–538], Robertson, Seymour and Thomas have discovered a finite structural description for every Kruskal ideal $\mathcal{I}$ (a set $\mathcal{I}$ of finite trees quasi-ordered and also closed under the well known topological minor embedding $\leq_t$). In this note, we generalize their result to finite trees that are vertex labeled from an arbitrary well-quasi-ordered (wqo) set and edge labeled from an ordinal, assuming the stronger relation known as the Kruskal-Friedman gap-embedding, "$\leq_{KF}$ ". We do not assume the result in the cited paper and so this serves as a new proof as well. The proof in the cited paper uses theoretical arguments for existence, whereas here a recursive algorithm that computes the finite structure is given. More importantly, this note lays a background theory that leads to simpler proofs to some of the deep theorems of Nash-Williams in [On well-quasi-ordering lower sets of finite trees. Proc. Cambridge Phil. Soc., 60:369-384, 1964], [On well-quasi-ordering infinite trees. Proc. Cambridge Philos. Soc. 61 (1965) 697-720], and [On better-quasi-ordering transfinite sequences. Proc. Cambridge Phil. Soc., 64:273-2090, 1968].

A simple proof of the [On well-quasi-ordering lower sets of finite trees] as a corollary of this result follows in [Y. Nigussie, On well-quasi-ordering lower sets of finite trees, a new proof, (in preparation)].

2008-881 Y. Nigussie
On well-quasi-ordering lower sets of finite trees, a new proof

In 1964, Nash-Williams proved in [On well-quasi-ordering lower sets of finite trees. Proc. Cambridge Phil. Soc., 60:369-384, 1964] that the iterated lower sets (ideals) of finite trees are well-quasi-ordered by the subset relation. However, acknowledges that his proof is very complicated. Robertson in 1997, [Y. Nigussie, N. Robertson, On Structural Description of lower Ideals of Tress, Journal of Graph Theory, 50(2005), 3 221-230] conjectured a "Lifting lemma" which claims that every ideal can be described by a finite tree in such way that an embedding relation between two description trees of two ideals lifts back to the subset relation between the corresponding two ideals.We prove the lifting conjecture in the affirmative. As a corollary, a simple proof of the result of [On well-quasi-ordering lower sets of finite trees] follows.

2008-882 J. Fiala, S. Klavžar, and B. Lidický
The packing chromatic number of infinite product graphs

The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k such that the vertex set $V(G)$ can be partitioned into disjoint classes $X_1, \ldots, X_k$, where vertices in X_i have pairwise distance greater than i. For the Cartesian product of a path and the 2-dimensional square lattice it is proved that $\chi_\rho(P_m \,\square\, \mathbb{Z}^2) = \infty$ for any $m \geq 2$, thus extending the result $\chi_\rho(\mathbb{Z}^3) = \infty$ of Finbow and Rall [On the packing chromatic number of some lattices, submitted to Discrete Appl. Math., special issue LAGOS'07]. It is also proved that $\chi_\rho(\mathbb{Z}^2) \geq 10$ which improves the bound $\chi_\rho(\mathbb{Z}^2) \geq 9$ of Goddard et al. [Broadcast chromatic numbers of graphs, Ars Combin. 86 (2008) 33–49]. Moreover, it is shown that $\chi_\rho(G \,\square\, \mathbb{Z}) < \infty$ for any finite graph G. The infinite hexagonal lattice $\mathcal{H}$ is also considered and it is proved that that $\chi_\rho(\mathcal{H}) \leq 7$ and $\chi_\rho(P_m \,\square\, \mathcal{H}) = \infty$ for $m \geq 6$.

2008-883 M. DeVos, L. Goddyn, B. Mohar, and R. Šámal
Cayley sum graphs and eigenvalues of $(3, 6)$-fullerenes

We determine the spectra of cubic plane graphs whose faces have sizes 3 and 6. Such graphs, "(3,6)-fullerenes", have been studied by chemists who are interested in their energy spectra. In particular we prove a conjecture of Fowler, which asserts that all their eigenvalues come in pairs of the form $\{\lambda, -\lambda\}$ except for the four eigenvalues $\{3, -1, -1, -1\}$. We exhibit other families of graphs which are "spectrally nearly bipartite" in the sense that nearly all of their eigenvalues come in pairs $\{\lambda, -\lambda\}$. Our proof utilizes a geometric representation to recognize the algebraic structure of these graphs, which turn out to be examples of Cayley sum graphs.

2008-884 J. Matoušek and R. Šámal
Induced trees in triangle-free graphs

We prove that every connected triangle-free graph on n vertices contains an induced tree on $\exp(c\sqrt{\log n}\,)$ vertices, where c is a positive constant. The best known upper bound is $(2+o(1))\sqrt{n}$. This partially answers questions of Erdős, Saks, and Sós and of Pultr.

2008-885 Z. Dvořák
Small Graph Classes and Bounded Expansion

A class of simple undirected graphs is *small* if it contains at most $n!\alpha^n$ graphs with n vertices, for some constant α. We prove that classes of graphs with expansion bounded by a function $f(r) = O(r^c)$ for any $0 \le c < \log_9 2 \approx 0.315$ are small.

2008-886 D. Král' and L. Stacho
Coloring plane graphs with independent crossings

We show that every plane graph with maximum face size four whose all faces of size four are vertex-disjoint is cyclically 5-colorable. This answers a question of Albertson whether graphs drawn in the plane with all crossings independent are 5-colorable.

2008-887 Z. Dvořák, K. Kawarabayashi, and R. Thomas
Three-coloring triangle-free planar graphs in linear time

Grötzsch's theorem states that every triangle-free planar graph is 3-colorable, and several relatively simple proofs of this fact were provided by Thomassen and other authors. It is easy to convert these proofs into quadratic-time algorithms to find a 3-coloring, but it is not clear how to find such a coloring in linear time (Kowalik used a nontrivial data structure to construct an $O(n \log n)$ algorithm).

We design a linear-time algorithm to find a 3-coloring of a given triangle-free planar graph. The algorithm avoids using any complex data structures, which makes it easy to implement. As a by-product we give another simple proof of Grötzsch's theorem.

2008-888 P. Kolman and O. Pangrác
On the Complexity of Paths Avoiding Forbidden Pairs

Given a graph $G = (V, E)$, two fixed vertices $s, t \in V$ and a set F of pairs of vertices (called *forbidden pairs*), the *problem of a path avoiding forbidden pairs* is to find a path from s to t that contains at most one vertex from each pair in F. The problem is known to be NP-complete in general and a few restricted versions of the problem are known to be in P. We study the complexity of the problem for directed acyclic graphs with respect to the structure of the forbidden pairs.

We write $x \prec y$ if and only if there exists a path from x to y and we assume, without loss of generality, that for every forbidden pair $(x, y) \in F$ we have $x \prec y$. The forbidden pairs have a *halving structure* if no two pairs $(u, v), (x, y) \in F$ satisfy $v \prec x$ or $v = x$ and they have a *hierarchical structure* if no two pairs $(u, v), (x, y) \in F$ satisfy $u \prec x \prec v \prec y$. We show that the PAFP problem is NP-hard even if the forbidden pairs have the halving structure and we provide a surprisingly simple and efficient algorithm for the PAFP problem with the hierarchical structure.

2008-889 V. Müller, J. Nešetřil, and V. Rödl
Some recollections on early work with Jan Pelant

In this note we consider three questions which can be traced to our early collaboration with Jan "Honza" Pelant. We present them from the contemporary perspective, in some cases extending our earlier work. The questions relate to Ramsey Theory, uniform spaces and tournaments.

2008-890 E. Lehtonen and J. Nešetřil
Minors of Boolean functions with respect to clique functions and hypergraph homomorphisms

Each clone $\mathcal{C}$ on a fixed base set A determines a quasiorder on the set of all operations on A by the following rule: f is a $\mathcal{C}$-minor of g if f can be obtained by substituting operations from $\mathcal{C}$ for the variables of g. By making use of a representation of Boolean functions by hypergraphs and hypergraph homomorphisms, it is shown that a clone $\mathcal{C}$ on $\{0, 1\}$ has the property that the corresponding $\mathcal{C}$-minor partial order is universal if and only if $\mathcal{C}$ is one of the countably many clones of clique functions or the clone of self-dual monotone functions. Furthermore, the $\mathcal{C}$-minor partial orders are dense when $\mathcal{C}$ is a clone of clique functions.

2008-891 J. Nešetřil and O. Serra

On a conjecture of Erdös and Turán for additive basis

An old conjecture of Erdős and Turán states that the representation function of an additive basis of the positive integers can not be bounded. We survey some results related to this still wide open conjecture.

2008-892 P. Hell and J. Nešetřil

Colouring, Constraint Satisfaction, and Complexity

Constraint satisfaction problems have enjoyed much attention since the early seventies, and in the last decade have become also a focus of attention amongst theoreticians. Graph colourings are a special class of constraint satisfaction problems; they offer a microcosm of many of the considerations that occur in constraint satisfaction. From the point of view of theory, they are well known to exhibit a dichotomy of complexity - the k-colouring problem is polynomial time solvable when $k \leq 2$, and NP-complete when $k \geq 3$. Similar dichotomy has been proved for the class of graph homomorphism problems, which are intermediate problems between graph colouring and constraint satisfaction. However, for general constraint satisfaction problems, dichotomy has only been conjectured. Although the conjecture remains unproven to this day, it has been driving much of the theoretical research on constraint satisfaction problems, which combines methods of logic, universal algebra, analysis, and combinatorics. Currently, this is a very active area of research, and it is our goal here to present some of the recent developments, updating some of the information in existing books and surveys, while focusing on both the mathematical and the computational aspects of the theory. Given the level of activity, we are only able to survey a fraction of the new work, with emphasis on our own areas of interest.

2008-893 D. Král', S. Norine, and O. Pangrác

Markov bases of binary graph models of K_4-minor free graphs

Markov width of a graph is a graph invariant defined as the maximum degree of a Markov basis element for the corresponding graph model for binary contingency tables. We show that a graph has Markov width at most four if and only if it contains no K_4 as a minor, answering a question of Develin and Sullivant. We also present a lower bound of order $\Omega(n^{2-\varepsilon})$ on the Markov width of K_n.

2008-894 Z. Dvořák, B. Lidický, and R. Škrekovski
Planar graphs without 3-, 7-, and 8-cycles are 3-choosable

A graph is k-choosable if it can be colored whenever every vertex has a list of available colors of size at least k. Grötzsch's theorem states that every planar triangle-free graph is 3-colorable. However, Voigt [A not 3-choosable planar graph without 3-cycles, *Discrete Math. 146 (1995) 325–328] gave an example of such a graph that is not 3-choosable, thus Grötzsch's theorem does not generalize naturally to choosability. We prove that every planar triangle-free graph without 7- and 8-cycles is 3-choosable.*

2008-895 T. Ebenlendr and J. Sgall
Semi-Online Preemptive Scheduling: One Algorithm for All Variants

We present a unified optimal semi-online algorithm for preemptive scheduling on uniformly related machines with the objective to minimize the makespan. This algorithm works for all types of semi-online restrictions, including the ones studied before, like sorted (decreasing) jobs, known sum of processing times, known maximal processing time, their combinations, and so on. Based on the analysis of this algorithm, we derive some global relations between various semi-online restrictions and tight bounds on the approximation ratios for a small number of machines.

2008-896 T. Ebenlendr and J. Sgall
A lower bound for scheduling of unit jobs with immediate decision on parallel machines

Consider scheduling of unit jobs with release times and deadlines on m identical machines with the objective to maximize the number of jobs completed before their deadlines. We prove a new lower bound for online algorithms with immediate decision. This means that the jobs arrive over time and the algorithm has to decide the schedule of each job immediately upon its release. Our lower bound tends to $e/(e-1) \approx 1.58$ for many machines, matching the performance of the best algorithm.

2008-897 D. Král', O. Serra, and L. Vena
A Removal Lemma for Systems of Linear Equations over Finite Fields

We prove a removal lemma for systems of linear equations over finite fields: let $X_1, \ldots, X_m$ be subsets of the finite field $\mathbb{F}_q$ and let A be a $(k \times m)$ matrix with coefficients in $\mathbb{F}_q$ and rank k; if the linear system $Ax = b$ has $o(q^{m-k})$ solutions with $x_i \in X_i$, then we can destroy all these solutions by deleting $o(q)$ elements from each X_i. This extends a result of Green [Geometric and Functional Analysis 15(2) (2005), 340–376] for a single linear equation in abelian groups to systems of linear equations. In particular, we also obtain an analogous result for systems of equations over integers, a result conjectured by Green. Our proof uses the colored version of the hypergraph Removal Lemma.

2008-898 J. Azarija, R. Erman, D. Král', M. Krnc, and L. Stacho
Cyclic colorings of plane graphs with independent faces

Let G be a plane graph with maximum face size Δ^*. If all faces of G with size four or more are vertex disjoint, then G has a cyclic coloring with $\Delta^* + 1$ colors, i.e., a coloring such that all vertices incident with the same face receive distinct colors.

2008-899 M. Klazar and M. Loebl (eds.)
Enumeration Workshop, Patejdlova bouda, Špindlerův Mlýn, November 17–23, 2007

2008-900 Z. Dvořák, B. Lidický, and R. Škrekovski
3-choosability of triangle-free planar graphs with constraint on 4-cycles

A graph is k-choosable if it can be colored whenever every vertex has a list of at least k available colors. A theorem by Grötzsch [Ein Dreifarbenzatz für dreikreisfreie netze auf der kugel, Math.-Natur. Reihe, 8:109–120, 1959] asserts that every triangle-free planar graph is 3-colorable. On the other hand Voigt [List colourings of planar graphs, Discrete Math., 120(1-3):215–219, 1993] gave such a graph which is not 3-choosable. We prove that every triangle-free planar graph such that 4-cycles do not share edges with other 4- and 5-cycles is 3-choosable. This strengthens the Thomassen's result [3-list-coloring planar graphs of girth 5, J. Combin. Theory Ser. B, 64(1):101–107, 1995] that every planar graph of girth at least 5 is 3-choosable. In addition, this implies that every triangle-free planar graph without 6- and 7-cycles is 3-choosable.

2008-901 Milan Hladík

Tolerance analysis in linear programming

We consider a linear programming problem and suppose that we have an optimal solution. In practice it is often important to know how different optimality criteria (optimal solution, optimal basis, optimal partition, etc.) changes when we perturb the input data. Our aim is to compute tolerances (intervals) for the objective function and right-hand side coefficients such that these coefficients can independently and simultaneously vary inside their tolerances while preserving the optimality criterion. We put the tolerance analysis in a unified framework that is convenient for algorithmic processing. We survey the known results (pioneered by R. E. Wendell) and propose an improvement that is optimal is some sense (the resulting tolerances are maximal and they take into account proportionality). We apply our approach to several sensitivity invariancies: optimal basis, support set and optimal partition invariancy. Thus the approach is convenient not only for simplex method solvers, but also for the interior points methods. We also discuss the time complexity and show that it is NP-hard to determine the maximal tolerances.

2008-902 P. Gregor and R. Škrekovski

On Generalized Middle Level Problem

Let G_n^k be the subgraph of the hypercube Q_n induced by levels between k and $n-k$, where $n \geq 2k+1$ is odd. The well-known middle level conjecture asserts that G_{2k+1}^k is Hamiltonian for all $k \geq 1$. We study this problem in G_n^k for fixed k. It is known that G_n^0 and G_n^1 are Hamiltonian for all odd $n \geq 3$. In this paper we prove that also G_n^2 is Hamiltonian for all odd $n \geq 5$, and we conjecture that G_n^k is Hamiltonian for every $k \geq 0$ and every odd $n \geq 2k+1$.

2008-903 D. Král', E. Máčajová, J. Mazák, and J. Sereni
Circular edge-colorings of cubic graphs with girth six

We show that the circular chromatic index of a (sub)cubic graph with girth at least six is at most $7/2$.

2008-904 O. Amini, L. Esperet, and J. van den Heuvel
A Unified Approach to Distance-Two Colouring of Graphs on Surfaces

In this paper we introduce the notion of (A, B)-colouring of a graph: For given vertex sets A, B, this is a colouring of the vertices in B so that both adjacent vertices and vertices with a common neighbour in A receive different colours. This concept generalises the notion of colouring the square of graphs and of cyclic colouring of graphs embedded in a surface. We prove a general result which implies asymptotic versions of Wegner's and Borodin's Conjecture on the planar version of these two colourings. Using a recent approach of Havet *et al.*, we reduce the problem to edge-colouring of multigraphs and then use Kahn's result that the list chromatic index is close to the fractional chromatic index.

Our results are based on a strong structural lemma for graphs embedded in a surface which also implies that the size of a clique in the square of a graph of maximum degree Δ embeddable in some fixed surface is at most $\frac{3}{2}\,\Delta$ plus a constant.

2008-905 L. Esperet, D. Král', P. Škoda, and R. Škrekovski
An improved linear bound on the number of perfect matchings in cubic graphs

We show that every cubic bridgeless graph with n vertices has at least $3n/4-10$ perfect matchings. This is the first bound that differs by more than a constant from the maximal dimension of the perfect matching polytope.

2008-906 T. Valla (ed.)
Abstracts of KAM-DIMATIA Series Year 2008
