Abstracts of KAM-DIMATIA Series Year 2006

Dear colleagues,

each year there are many interesting submissions in KAM-DIMATIA Series, much more than is possible to read by a mere human. To help people overview what is going on at KAM and DIMATIA in general, we prepared a list of abstracts of all papers submitted each year. We hope you will enjoy reading it.

Martin Bálek
editor of KAM-DIMATIA Series

2006-759 I. Moffatt
Alternating Links are Non-Trivial

We use a simple geometric argument and small cancellation properties of link groups to prove that alternating links are non-trivial. This proof uses only classic results in topology and combinatorial group theory.

2006-760 M. Hladík
Separation of two convex polyhedral sets with parameters in one column of the constraint matrix

Separation is a famous principle and separation properties are important for optimization theory and various applications. In practice, input data are rarely known exactly and it is advisable to deal with parameters. In this article, we are concerned with the basic characteristics (existence, description, stability etc.) of separating hyperplanes of two convex polyhedral sets depending on parameters. We study the case, when parameters are situatedin one column of the constraint matrix from the description of the given convex polyhedral set.

2006-761 M. Hladík
Separation of two convex polyhedral sets with parameters in one row of the constraint matrix

In this paper we study the separation properties of two convex polyhedral sets for the case there are parameters in one row of the constraint matrix. Especially, we deal with the existence, description and stability properties of the separating hyperplanes of such convex polyhedral sets. Briefly, we are also interested in supporting separation (separating hyperplanes supports convex polyhedral sets at given faces).

2006-762 J. Fiala, D. Paulusma, and J. A. Telle
Locally constrained graph homomorphisms and equitable partitions

We explore the connection between locally constrained graph homomorphisms and degree matrices arising from an equitable partition of a graph. We provide several equivalent characterizations of degree matrices. As a consequence we can efficiently check whether a given matrix M is a degree matrix of some graph and also compute the size of a smallest graph for which it is a degree matrix in polynomial time. We extend the well-known connection between degree *refinement* matrices of graphs and locally bijective graph homomorphisms to locally injective and locally surjective homomorphisms by showing that also these latter types of homomorphisms impose a quasiorder on degree matrices and a partial order on degree refinement matrices. Computing the degree refinement matrix of a graph is easy, and an algorithm deciding comparability of two matrices in one of these partial orders could be used as a heuristic for deciding whether a graph G allows a homomorphism of the given type to H. For local surjectivity and injectivity we show that the problem of matrix comparability belongs to the complexity class NP.

Keywords: locally constrained graph homomorphism, partial order, degree matrix, computational complexity.

2000 Mathematics Subject Classification: 05C15, 06A07, 03D15.

2006-763 J. Nešetřil and P. Ossona de Mendez

Linear time low tree-width partitions and consequences

Classes of graphs with bounded expansion generalize both proper minor closed classes and classes with bounded degree.

For any class with bounded expansion $\mathcal{C}$ and any integer p there exists a constant $N(\mathcal{C}, p)$ so that the vertex set of any graph $G \in \mathcal{C}$ may be partitioned into at most $N(\mathcal{C}, p)$ parts, any $i \leq p$ parts of them induce a subgraph of tree-width at most $(i-1)$ (actually, of *tree-depth* at most i, what is sensibly stronger). Such partitions are central to the resolution of homomorphism problems like *restricted homomorphism dualities*.

We give here a simple algorithm to compute such partitions and prove that if we restrict the input graph to some fixed class $\mathcal{C}$ with bounded expansion, the running time of the algorithm is bounded by a linear function of the order of the graph (for fixed $\mathcal{C}$ and p).

This result is applied to get a linear time algorithm for the subgraph isomorphism problem with fixed pattern and input graphs in a fixed class with bounded expansion.

More generally, let ϕ be a first order logic sentence. We prove that any fixed graph property of type

$$\text{“ } \exists X : (|X| \leq p) \wedge (G[X] \vDash \phi) \text{ ”}$$

may be decided in linear time for input graphs in a fixed class with bounded expansion.

2006-764 J. Foniok, J. Nešetřil and C. Tardif

Generalised Dualitiesand Finite Maximal Antichains

We fully characterise the situations where the existence of a homomorphism from a digraph G to at least one of a finite set $\mathcal{H}$ of directed graphs is determined by a finite number of forbidden subgraphs. We prove that these situations, called *generalised dualities*, are characterised by the non-existence of a homomorphism to G from a finite set of forests.

Furthermore, we characterise all finite maximal antichains in the partial order of directed graphs ordered by the existence of homomorphism. We show that these antichains correspond exactly to the generalised dualities. This solves a problem posed by Nešetřil and Tardif in 2003. Finally, we show that it is NP-hard to decide whether a finite set of digraphs forms a maximal antichain.

2006-765 J. Foniok and J. Nešetřil (eds.)
DIMATIA Annual Report 2005 (10 years of DIMATIA)

2006-766 J. Foniok, J. Nešetřil and C. Tardif
Generalised dualities and maximal finite antichains in the homomorphism order of relational structures

The motivation for this paper is three-fold. First, we study the connectivity properties of the homomorphism order of directed graphs, and more generally for relational structures. As opposed to the homomorphism order of undirected graphs (which has no non-trivial finite maximal antichains), the order of directed graphs has finite maximal antichains of any size. In this paper, we characterise explicitely all maximal antichains in the homomorphism order of directed graphs.

Quite surprisingly, these maximal antichains correspond to generalised dualities. The notion of generalised duality is defined here in the full generality as an extension of the notion of finitary duality, investigated by Nešetřil and Tardif, 2000. Building upon the results of the cited paper, we fully characterise the generalised dualities. It appears that these dualities are determined by forbidding homomorphisms from a finite set of forests (rather than trees).

Finally we shall characterise "generalised" Constraint Satisfaction Problems (defined also here) problems that are first order definable. These are again just generalised dualities corresponding to finite maximal antichains in the homomorphism order.

2006-767 M. Loebl and I. Moffatt
The chromatic polynomial of fatgraphs and its categorification

We introduce a homology theory for embedded graphs whose graded Euler characteristic is the chromatic polynomial, and whose Poincaré polynomial is invariant on different planar embeddings of the same graph. We conjecture that the homology can detect the genus of the embedding. We show that for planar embedded graphs our homology is obtained by 'adding coefficients' to the Khovanov's categorification of the Jones polynomial. At present there are extensive efforts by mathematicians and physicists to extend the Khovanov homology theory. Both the chromatic polynomial and the Jones polynomial are determined by the Tutte polynomial on the (signed) graphs, or equivalently by the $+-J$ Potts partition function. We extend this to the homology: we categorify the Bollobás-Riordan topological Tutte polynomial of signed embedded graphs so that the homology of the chromatic polynomial can be recovered by a projection and the Khovanov homology theory of the Jones polynomial can be recovered using a 'universal coefficients theorem'.

2006-768 R. N. Ball, A. Pultr, and J. Sichler
More on Configurations in Priestley Spaces

Prohibiting configurations ($\equiv$ induced finite connected posets) in Priestley spaces and properties of the associated classes of distributive lattices, and the related problem of configurations in coproducts of Priestley spaces, have been brought to satisfactory conclusions in case of configurations with a unique maximal element. The general case is, however, far from settled. After a short survey of known results we present the desired answers for a large (although still not complete) class of configurations without top.

2006-769 J. Picado and A. Pultr
Sublocale sets and sublocale lattices

We present very short and simple proofs of such facts as co-frame distributivity of sublocales, zero-dimensionality of the resulting co-frames, Isbell's Density Theorem and characteristic properties of fit and subfit frames, using sublocale sets.

2006-770 J. Kára (ed.)
Midsummer Combinatorial Workshop 2005 and
DIMACS, DIMATIA, Rényi Workshop 2005

2006-771 I. Moffatt
Knot Invariants and the Bollobás-Riordan Polynomial

For a graph G embedded in an orientable surface Σ, we consider associated links $\mathcal{L}(G)$ in the thickened surface $\Sigma \times I$. We relate the HOMFLY polynomial of $\mathcal{L}(G)$ to the recently defined Bollobás-Riordan polynomial of a ribbon graph. This generalizes celebrated results of Jaeger and Traldi. We use knot theory to prove results about graph polynomials and, after discussing questions of equivalence of the polynomials, we go on to use our formulae to prove a duality relation for the Bollobás-Riordan polynomial. We then consider the specialization to the Jones polynomial and recent results of Chmutov and Pak to relate the Bollobás-Riordan polynomials of an embedded graph and its tensor product with a cycle.

2006-772 R. Šámal
On XY mappings $\star$ Tension-continuous and related types of mappings

This is a treatise on certain mappings between graphs, defined by means of cycle structure of the respective graphs. We study these mappings from various perspectives: we start by comparing them to graph homomorphisms, this may be viewed as a type of reconstruction problems, as we study to what extent is a graph determined by its cycle structure. We also take the point of view of category theory and study the structure that these mappings impose on the class of all graphs. Next, we get to more applicable aspects of the mappings under study:

- We use them to prove certain relaxation of Pentagon Problem due to Nešetřil.

- We introduce a new graph invariant which promises to be useful more generally for study of graph homomorphisms.
- We use our mappings to bring new understanding to various conjectures concerning cycle structure of graphs (particularly to Cycle double cover conjecture).

2006-773 J. Hladký, M. Krčál and B. Lidický (eds.)
Spring School on Combinatorics 2006

2006-774 D. Král' and R. Thomas
Coloring even-faced graphs in the torus and the Klein bottle

We prove that a triangle-free graph drawn in the torus with all faces bounded by even walks is 3-colorable if and only if it has no subgraph isomorphic to the Cayley graph $C(Z_{13}; 1, 5)$. We also prove that a non-bipartite quadrangulation of the Klein bottle is 3-colorable if and only if it has no non-contractible separating cycle of length at most four and no odd walk homotopic to a non-contractible two-sided simple closed curve. These results settle a conjecture of Thomassen and two conjectures of Archdeacon, Hutchinson, Nakamoto, Negami and Ota.

2006-775 G. Kun and J. Nešetřil
Forbidden Lifts(NP and CSP for combinatorists)

We present a definition of the class NP in combinatorial context as the class of languages of structures defined by finitely many forbidden lifted substructures. We apply this to special syntactically defined subclasses and show how they correspond to naturally defined (and intensively studied) combinatorial problems. We show that some types of combinatorial problems like edge colorings and graph decompositions express the full computational power of the class NP. We then characterize Constraint Satisfaction Problems (i.e. H-coloring problems) which are expressible by finitely many forbidden lifted substructures. This greatly simplifies and generalizes the earlier attempts to characterize this problem. As a corollary of this approach we perhaps find a proper setting of Feder and Vardi analysis of CSP languages within the class MMSNP.

2006-776 P. Kolman and T. Waleń
Reversal Distance for Strings with Duplicates: Linear Time Approximation using Hitting Set

In the last decade there has been an ongoing interest in string comparison problems; to a large extend the interest was stimulated by genome rearrangement problems in computational biology but related problems appear in many other areas of computer science. Particular attention has been given to the problem of *sorting by reversals* (SBR): given two strings, A and B, find the minimum number of reversals that transform the string A into the string

B (a *reversal* $\rho(i,j)$, $i < j$, transforms a string $A = a_1 \dots a_n$ into a string $A' = a_1 \dots a_{i-1} a_j a_{j-1} \dots a_i a_{j+1} \dots a_n$).

Primarily the problem has been studied for strings in which every symbol appears exactly once (that is, for permutations) and only recently attention has been given to the general case where duplicates of the symbols are allowed. In this paper we consider the problem k-SBR, a version of SBR in which each symbol is allowed to appear up to k times in each string, for some $k \geq 1$. The main result of the paper is a $\Theta(k)$-approximation algorithm for k-SBR running in time $O(n)$; compared to the previously known algorithm for k-SBR, this is an improvement by a factor of $\Theta(k)$ in the approximation ratio, and by a factor of $\Theta(k)$ in the running time. Crucial ingredients of our algorithm are the suffix tree data structure a linear time algorithm for a special case of a disjoint set union problem.

2006-777 M. Erné, M. Gehrke, and A. Pultr
Complete Congruences on Topologies and Down-Set Lattices

From the work of Simmons about nuclei in frames it follows that a topological space X is scattered if and only if each congruence Θ on the frame of open sets is induced by a unique subspace A so that $\Theta = \{(U, V) \mid U \cap A = V \cap A\}$, and that the same holds without the uniqueness requirement iff X is weakly scattered (corrupt). We prove a seemingly similar but substantially different result about quasidiscrete topologies (in which arbitrary intersections of open sets are open): each complete congruence on such a topology is induced by a subspace if and only if the corresponding poset is (order) scattered, i.e. contains no dense chain. More questions concerning relations between frame, complete, spatial, induced and open congruences are discussed as well.

2006-778 J. Nešetřil
Homomorphisms of Structures (concepts and highlights)

In this paper we survey the recent results on graph homomorphisms perhaps for the first time in the broad range of their relationship to wide range applications in computer science, physics and other branches of mathematics. We illustrate this development in each area by few results.

2006-779 J. R. Griggs and D. Král'
Graph Labellings with Variable Weights, a Survey

Graph labellings form an important graph theory model for the channel assignment problem. An optimum labelling usually depends on one or more parameters that ensure minimum separations between frequencies assigned to nearby transmitters. The study of spans and of the structure of optimum labellings as functions of such parameters has attracted substantial attention from researchers, leading to the introduction of real number graph labellings and λ-graphs. We survey recent results obtained in this area.

2006-780 J. Nešetřil and R. Šámal
On tension-continuous mappings

Tension-continuous (shortly TT) mappings are mappings between the edge sets of graphs. They generalize graph homomorphisms. From another perspective, tension-continuous mappings are dual to the notion of flow-continuous mappings and the context of nowhere-zero flows motivates several questions considered in this paper.

Extending our earlier research we define new constructions and operations for graphs (such as graphs $\Delta_M(G)$) and give evidence for the complex relationship of homomorphisms and TT mappings. Particularly, solving an open problem, we display pairs of TT-comparable and homomorphism-incomparable graphs with arbitrarily high connectivity.

We give a new (and more direct) proof of density of TT order and study graphs such that TT mappings and homomorphisms from them coincide; we call such graphs homotens. We show that most graphs are homotens, on the other hand every vertex of a nontrivial homotens graph is contained in a triangle. This provides a justification for our construction of homotens graphs.

2006-781 M. DeVos and R. Šámal
High Girth Cubic Graphs Map to the Clebsch Graph

We give a (computer assisted) proof that the edges of every graph with maximum degree 3 and girth at least 17 may be 5-colored (possibly improperly) so that the complement of each color class is bipartite. Equivalently, every such graph admits a homomorphism to the Clebsch graph.

Hopkins and Staton, 1982 and Bondy and Locke, 1986, proved that every (sub)cubic graph of girth at least 4 has an edge-cut containing at least $\frac{4}{5}$ of the edges. The existence of such an edge-cut follows immediately from the existence of a 5-edge-coloring as described above, so our theorem may be viewed as a kind of coloring extension of their result (under a stronger girth assumption).

Every graph which has a homomorphism to a cycle of length five has an above-described 5-edge-coloring; hence our theorem may also be viewed as a weak version of Nešetřil's Pentagon Problem: Every cubic graph of sufficiently high girth maps to C_5.

2006-782 M. Klazar
Introduction to Number Theory(lecture notes)

2006-783 M. Bodirsky and D. Král'
Locally Consistent Constraint Satisfaction Problems

An instance of a constraint satisfaction problem (CSP) is *variable k-consistent* if any subinstance with at most k variables has a solution. For a fixed constraint language $\mathcal{L}$, $\rho_k(\mathcal{L})$ is the largest ratio such that any variable k-consistent instance has a solution that satisfies at least a fraction of $\rho_k(\mathcal{L})$ of the constraints.

We provide an expression for the limit $\rho(\mathcal{L}) := \lim_{k\to\infty} \rho_k(\mathcal{L})$, and show that this limit coincides with the corresponding limit for *constraint k-consistent* instances, i.e., instances where all subinstances with at most k constraints have a solution. We also design an algorithm that for an input instance and a given ε either computes a solution that satisfies at least a fraction of $\rho(\mathcal{L}) - \varepsilon$ constraints or finds a set of inconsistent constraints whose size only depends on ϵ. Most of our results apply both to weighted and to unweighted instances of the constraint satisfaction problem.

2006-784 R. N. Ball, J. Nešetřil and A. Pultr
Dualities in full homomorphisms

In this paper we study dualities of graphs and, more generally, relational structures with respect to full homomorphisms, that is, mappings that are both edge- and non-edge-preserving. The research was motivated, a.o., by results from logic (concerning first order definability) and Constraint Satisfaction Problems. We prove that for any finite set of objects $\mathcal{B}$ (finite relational structures) there is a finite duality with $\mathcal{B}$ to the left. It appears that surprising richness of these dualities leads to interesting problems of Ramsey type; they are which are explicitly analyzed in the simplest case of graphs.

2006-785 M. Hladík
Linear interval systems with a specific dependence structure

This is a contribution to solvability of linear interval equations and inequalities. In interval analysis we usually suppose that values from different intervals are mutually independent. This assumption can be sometimes too restrictive. In this article we derive extensions of Oettli–Prager theorem and Gerlach theorem for the case there is a simple dependence structure between coefficients of an interval system. The dependence is given by equality of two submatrices of the constraint matrix.

2006-786 D. Král'
Computing representations of matroids of bounded branch-width

For every $k \geq 1$ and two finite fields $\mathbb{F}$ and $\mathbb{F}'$, we design a polynomial-time algorithm that given a matroid $\mathcal{M}$ of branch-width at most k represented over $\mathbb{F}$ decides whether $\mathcal{M}$ is representable over $\mathbb{F}'$ and if so, it computes a representation of $\mathcal{M}$ over $\mathbb{F}'$. The algorithm also counts the number of non-isomorphic representations of $\mathcal{M}$ over $\mathbb{F}'$. Moreover, it can be modified to list all such non-isomorphic representations.

2006-787 D. Král', ed.
International REU 1999–2006

2006-788 H. Bruhn, J. Černý, A. Hall and P. Kolman

Single Source Multiroute Flows and Cuts on Uniform Capacity Networks

For an integer $h \geq 1$, an *elementary h-route flow* is a flow along h edge disjoint paths between a source and a sink, each path carrying a unit of flow, and a single commodity *h-route flow* is a non-negative linear combination of elementary h-route flows. An instance of a *single source multicommodity flow problem* for a graph $G = (V, E)$ consists of a source vertex $s \in V$ and k sinks $t_1, \ldots, t_k \in V$; we denote it $\mathcal{I} = (s; t_1, \ldots, t_k)$. In the *single source multicommodity multiroute flow problem*, we are given an instance $\mathcal{I} = (s; t_1, \ldots, t_k)$ and an integer $h \geq 1$, and the objective is to maximize the total amount of flow that is transferred from the source to the sinks so that the capacity constraints are obeyed and, moreover, the flow of each commodity is an h-route flow.

We study the relation between classical and multiroute single source flows on networks with uniform capacities and we provide a tight bound. In particular, we prove the following result. Given an instance $\mathcal{I} = (s; t_1, \ldots, t_k)$ such that each $s - t_i$ pair is h-connected, the maximum classical flow between s and the t_i's is at most $2(1 - 1/h)$-times larger than the maximum h-route flow between s and the t_i's and this is the best possible bound for $h \geq 2$. This, as we show, is in contrast to the situation of general multicommodity multiroute flows that are up to $k(1 - 1/h)$-times smaller than their classical counterparts.

As a corollary, we establish a max-flow min-cut theorem for the single source multicommodity multiroute flow and cut. An *h-disconnecting cut* for $\mathcal{I}$ is a set of edges $F \subseteq E$ such that for each i, the maximum h-route flow between $s - t_i$ is zero. We show that the maximum h-route flow is within $2(h - 1)$ of the mininimum h-disconnecting cut, independently of the number of commodities; we also describe a $2(h - 1)$-approximation algorithm for the minimum h-disconnecting cut problem.

2006-789 Mark H. Siggers

On the Bounded Degree Restriction of Constraint Satisfaction Problems

Given a graph H, let $b(H)$ be the minimum integer b, if it exists, for which H-colouring is NP-complete when restricted to instances with degree bounded by b. We show for any loopless non-bipartite graphs H that $b(H)$ is bounded above by a function of the size of H. Futhermore, we get tight upper bounds on $b(H)$ for various H. For example, we show that $b(H) = 3$ for any graph H with girth at least 7 in which every edge lies in a g-cycle, where g is the odd-girth of H.

Extending our purview to CSPs, we show for relational systems $(\mathcal{H})$ with projective cores $\mathcal{C}$, that the analagously defined function $b(\mathcal{H})$ is bounded above by a polynomial function of the maximum degree of $\mathcal{C}$.

2006-790 R. Erman, S. Jurečič, D. Král', K. Stopar, and N. Stopar

Optimal real number graph labelings of a subfamily of Kneser graphs

A notion of real number graph labelings captures the dependence of the span of an optimal channel assignment on the separations that are required between frequencies assigned to close transmitters. We determine the spans of such optimal labelings for a subfamily of Kneser graphs formed by the complements of the line graphs of complete graphs. This subfamily contains (among others) the Petersen graph.

2006-791 J. Fiala and J. Soto

Block transitivity and degree matrices

We say that a square matrix $\mathbf{M}$ of order r is a degree matrix of a given graph G if there is a so called equitable partition of its vertices into r blocks. This partition satisfies that for any i and j it holds that a vertex from the i-th block of the partition has exactly $m_{i,j}$ neighbors inside the j-th block.

We ask whether for a given degree matrix $\mathbf{M}$, there exists a graph G such that $\mathbf{M}$ is a degree matrix of G, and in addition, for any two edges e, f spanning between the same pair of blocks there exists an automorphism of G that sends e to f. In this work, we affirmatively answer the question for all degree matrices and show a way to construct a graph that witness this fact.

We further explore a case where the automorphism is required to exchange given pair of edges and show some positive and negative results.

2006-792 Z. Dvořá, D. Král', and J. Teska

Toughness threshold for the existence of 2-walks in K_4-minor free graphs

We show that every K_4-minor free graph with toughness greater than 4/7 has a 2-walk, i.e., a closed walk visiting each vertex at most twice. We also give an example of a 4/7-tough K_4-minor free graph with no 2-walk.

2006-793 P. Hell and J. Nešetřil

On the density of trigraph homomorphisms

An order is dense if $A < B$ implies $A < C < B$ for some C. The homomorphism order of (nontrivial) graphs is known to be dense. Homomorphisms of trigraphs extend homomorphisms of graphs, and model many partitions of interest in the study of perfect graphs. We address the question of density of the homomorphism order for trigraphs. It turns out that there are gaps in the order, and we exactly characterize where they occur.

2006-794 D. Král' and P. Nejedlý

Distance Constrained Labelings of K_4-minor Free Graphs

Motivated by previous results on distance constrained labelings and coloring of squares of K_4-minor free graphs, we show that for every $p \geq q \geq 1$, there exists Δ_0 such that every K_4-minor free graph G with maximum degree $\Delta \geq \Delta_0$ has an $L(p,q)$-labeling of span at most $q\lfloor 3\Delta(G)/2 \rfloor$. The obtained bound is the best possible.

2006-795 Z. Dvořák, S. Jendrol', and D. Král'
Matchings and non-rainbow colorings

We show that the maximum number of colors that can be used in a vertex coloring of a cubic 3-connected plane graph G that avoids a face with vertices of mutually distinct colors (a rainbow face) is equal to $\frac{n}{2} + \mu^* - 2$ where n is the number of vertices of G and μ^* is the size of the maximum matching of the dual graph G^*.

2006-796 M. DeVos, L. Goddyn, B. Mohar, and R. Šámal
A quadratic lower bound for subset sums

Let A be a finite nonempty subset of an additive abelian group G, and let $\Sigma(A)$ denote the set of all group elements representable as a sum of some subset of A. We prove that $|\Sigma(A)| \geq |H| + \frac{1}{64}|A \setminus H|^2$ where H is the stabilizer of $\Sigma(A)$. Our result implies that $\Sigma(A) = \mathbb{Z}/n\mathbb{Z}$ for every set A of units of $\mathbb{Z}/n\mathbb{Z}$ with $|A| \geq 8\sqrt{n}$. This consequence was first proved by Erdős and Heilbronn for n prime, and by Vu (with a weaker constant) for general n.

2006-797 T. Müller and J.-S. Sereni
Identifying codes in (random) geometric networks

It is important for networks built from wireless sensors technology to have a functional location detection system. Identifying codes, first introduced to model fault diagnosis of multi-processor systems have recently proved to be useful to address this question. We are interested in the situation where the area of communication of each sensor is modelled by a disk: thus we consider identifying codes for the class of unit disk graphs. Minimising the size of an identifying code is $\mathcal{NP}$-complete even for bipartite graphs. First, we improve this result by showing that the problem remains $\mathcal{NP}$-complete for bipartite planar unit disk graphs. Then, we address the question of the existence of an identifying code for random unit disk graphs. From a practical point of view, this corresponds to the case when sensors are randomly thrown on a plane. We derive the probability that there exists an identifying code as a function of the radius of the disks. The results obtained are in sharp contrast with those concerning random graphs in the Erdős and Rényi model.

2006-798 B. Lužar, R. Škrekovski, and M. Tancer
Injective colorings of planar graphs with few colors

An injective coloring of a graph is a vertex coloring where two vertices have distinct colors if a path of length two exists between them. In this paper some results on injective colorings of planar graphs with few colors are presented. We show that all planar graphs of girth ≥ 19 and maximum degree Δ are injectively Δ-colorable. We also show that all planar graphs of girth ≥ 10 are injectively $(\Delta+1)$-colorable, $\Delta+4$ colors are sufficient for planar graphs of girth ≥ 5 if Δ is large enough, and that subcubic planar graphs of girth ≥ 7 are injectively 5-colorable.

2006-799 J. Matoušek and A. Přívětivý

Large Monochromatic Components in Two-colored Grids

Let D_n^d denote the d-dimensional *grid with diagonals*, that is, the graph with vertex set $\{1, 2, \ldots, n\}^d$ and with edges connecting every two vertices that differ by at most 1 in every coordinate. We prove that for an arbitrary coloring of the vertices of D_n^d by two colors there exists a monochromatic connected subgraph with at least $n^{d-1} - d^2 n^{d-2}$ vertices, and thus the "horizontal layer" coloring (by the parity of the first coordinate) is almost optimal.

We also consider a d-dimensional *triangulated grid*; this is the graph of a triangulation of the solid cube $[1, n]^d$ that refines the subdivision of $[1, n]^d$ into the grid of unit cubes. Here every 2-coloring has a monochromatic connected subgraph with $\Omega(n^{d-1}/\sqrt{d})$ vertices.

These results are proved by combining combinatorial and topological arguments with suitable isoperimetric inequalities, and they can be viewed as d-dimensional generalizations of the planar HEX lemma.

2006-800 D. Král' and M. Stehlík

Coloring of triangle-free graphs on the double torus

We show that every triangle-free graph on the double torus is 4-colorable. This settles a problem raised by Gimbel and Thomassen [Trans. Amer. Math. Soc. 349 (1997), 4555–4564].
