Abstracts of KAM-DIMATIA Series Year 2005

Dear colleagues,

each year there are many interesting submissions in KAM-DIMATIA Series, much more than is possible to read by a mere human. To help people overview what is going on at KAM and DIMATIA in general, we prepared a list of abstracts of all papers submitted each year. We hope you will enjoy reading it.

Martin Bálek
editor of KAM-DIMATIA Series

2005-720 D. Kráľ, R. Škrekovski, and M. Tancer
Construction of large graphs with no optimal surjective $L(2,1)$-labelings

An $L(2,1)$-labeling of a graph G is a mapping $c : V(G) \to \{0, \dots, K\}$ such that the labels of two adjacent vertices differ by at least two and the labels of vertices at distance two differ by at least one. A hole of c is an integer $h \in \{0, \dots, K\}$ that is not used as a label for any vertex of G. The smallest integer K for which an $L(2,1)$-labeling of G exists is denoted by $\lambda(G)$. The minimum number of holes in an optimal labeling, i.e., a labeling with $K = \lambda(G)$, is denoted by $\rho(G)$. Georges and Mauro showed that $\rho(G) \le \Delta$, where Δ is the maximum degree of G, and conjectured that if $\rho(G) = \Delta$ and G is connected, then the order of G is at most $\Delta(\Delta + 1)$. We disprove this conjecture by constructing graphs G with $\rho(G) = \Delta$ and order $\left\lfloor \frac{(\Delta+1)^2}{4} \right\rfloor (\Delta + 1) \approx \Delta^3/4$.

2005-721 J. Fox, V. Jungić, and R. Radoičić
Sub-Ramsey numbers for Arithmetic Progressions and Schur Triples

For a given positive integer k, $sr(m,k)$ denotes the minimal positive integer such that every coloring of $[n]$, $n \ge sr(m,k)$, that uses each color at most k times, yields a rainbow $AP(m)$; that is, an m-term arithmetic progression, all of whose terms receive different colors. We prove that $\frac{17}{8}k + O(1) \le sr(3,k) \le \frac{15}{7}k + O(1)$ and $sr(m,2) > \lfloor \frac{m^2}{2} \rfloor$, improving the previous bounds of Alon, Caro, and Tuza from 1989. Our new lower bound on $sr(m,2)$ immediately implies that for $n \le \frac{m^2}{2}$, there exists a mapping $\phi : [n] \to [n]$ without a fixed point such that for every $AP(m)$ $\mathcal{A}$ in $[n]$, the set $\mathcal{A} \cap \phi(\mathcal{A})$ is not empty. We also propose the study of sub-Ramsey–type problems for linear equations other than $x + y = 2z$. For a given positive integer k, we define $ss(k)$ to be the minimal positive integer n such that every coloring of $[n]$, $n \ge ss(k)$, that uses each color at most k times, yields a rainbow solution to the Schur equation $x + y = z$. We prove that $ss(k) = \lfloor \frac{5k}{2} \rfloor + 1$.

2005-722 D. Kráľ, J. Sgall, and T. Tichý
Randomized Strategies for the Plurality Problem

We consider a game played by two players, Paul and Carol. At the beginning of the game, Carol fixes a coloring of n balls. At each turn, Paul chooses a pair of the balls and asks Carol whether the balls have the same color. Carol truthfully answers his question. Paul's goal is to determine the most frequent (plurality) color in the coloring by asking as few questions as possible. The game is studied in the probabilistic setting when Paul is allowed to choose his next question randomly.

We give asymptotically tight bounds both for the case of two colors and many colors. For the balls colored by two colors, we provide a strategy for Paul to determine the plurality color with the expected number of $2n/3 + O(\sqrt{n \log n})$ questions and a lower bound $2n/3 - O(\sqrt{n})$ on the expected number of Paul's questions. For the balls colored by k colors, we prove a lower bound $\Omega(kn)$ on the expected number of questions.

2005-723 J. Fiala, P. Golovach, and J. Kratochvíl
Distance constrained labelings of graphs of bounded treewidth

We prove that the L(2,1)-LABELING problem is NP-complete for graphs of treewidth two, thus adding a natural and well studied problem to the short list of problems whose computational complexity separates treewidth one from treewidth two. We prove similar results for other variants of the distance constrained graph labeling problem.

2005-724 M. Bodirsky and D. Král'
Locally Consistent Constraint Satisfaction Problems with Binary Constraints

An instance of a constraint satisfaction problem is k-consistent if any k constraints of it can be simultaneously satisfied. We focus on constraint languages with a single binary constraint. In this case, the constraint satisfaction problem is equivalent to the question whether there is a homomorphism from an input digraph G to a fixed target digraph H. The instance corresponding to G is k-consistent if every subgraph of G of size at most k is homomorphic to H. Let $\rho_k(H)$ be the largest ρ such that every k-consistent G contains a subgraph G' of size at least $\rho||E(G)||$ that is homomorphic to H. The ratio $\rho_k(H)$ reflects the fraction of constraints of a k-consistent instance that can be always satisfied. We determine $\rho_k(H)$ for all digraphs H that are not acyclic and show that $\lim_{k\to\infty} \rho_k(H) = 1$ if H has tree duality. For the latter case we design an efficient algorithm that computes in linear time for a given input graph G and $\varepsilon > 0$ either a homomorphism from almost entire graph G to H or a subgraph of G of bounded size that is not homomorphic to H.

2005-725 M. Hladík
Separation properties of two convex polyhedral sets with RHS-parameters

Separation of convex sets is widely used in many branches of mathematics. Often in practice input data are known only approximately and it is advisable to deal with parameters. We initiate a combining of these two principles – separation and parametrization – together. Important properties (existence, description, stability etc.) of separating hyperplanes of two convex polyhedral sets depending on right-hand-side parameters, is dealt with in this article.

2005-726 D. Král', L. Tong, and X. Zhu
Upper Hamiltonian Numbers and Hamiltonian Spectra of Graphs

If π is a cyclic order of the vertices of a graph G, the number $h(\pi)$ is defined to be the sum of the distances between consecutive vertices of G in π. For a graph G, the hamiltonian spectrum $\mathcal{H}(G)$ is the set of all numbers $h(\pi)$. The hamiltonian number $h(G)$ of G is the minimum number contained in $\mathcal{H}(G)$ and the upper hamiltonian number $h^+(G)$ is the maximum number contained in $\mathcal{H}(G)$. We determine hamiltonian spectra of cycles. We also show that the upper hamiltonian number of a graph G of order n and diameter d is at least $n + \lceil d^2/2 \rceil - 1$. The bound is tight for all pairs n and d.

2005-727 Z. Dvořák, D. Král', P. Nejedlý, and R. Škrekovski
Coloring squares of planar graphs with no short cycles

Wang and Lih conjectured that for every $g \geq 5$, there exists a number $M(g)$ such that the chromatic number of the square of every planar graph of girth at least g and maximum degree $\Delta \geq M(g)$ is $\Delta + 1$. We disprove the conjecture for $g \in \{5, 6\}$ and prove the existence of the number $M(g)$ for $g \geq 7$. More generally, we show that every planar graph of girth at least 7 and maximum degree $\Delta \geq 190 + 2\lceil p/q \rceil$ has an $L(p, q)$-labeling of span at most $2p + q\Delta - 2$. For $q = 1$, the bound is tight for all pairs of Δ and p. We also show that the square of every planar graph of girth at least six and sufficiently large maximum degree Δ is $(\Delta + 2)$-colorable.

2005-728 M. Bodirsky and J. Nešetřil
Constraint Satisfaction with Countable Homogeneous Templates

For a fixed countable homogeneous relational structure Γ we study the computational problem whether a given finite structure of the same signature homomorphically maps to Γ. This problem is known as the *constraint satisfaction problem* CSP(Γ) for the *template* Γ and was intensively studied for finite Γ. We show that – as in the case of finite Γ – the computational complexity of CSP(Γ) for countable homogeneous Γ is determinded by the clone of polymorphisms of Γ. To this end we prove the following theorem, which is of independent interest: The primitive positive definable relations over an ω-categorical structure Γ are precisely the relations that are preserved by the polymorphisms of Γ. If the age of Γ is given by a finite number of finite forbidden induced substructures, then CSP(Γ) is in NP. We use a classification result by Cherlin and prove that in this case every constraint satisfaction problem for a countable homogeneous digraph is either tractable or NP-complete.
Keywords: complexity of constraint satisfaction, homogeneous digraphs, graph homomorphisms, ω-categorical structures, polymorphism preservation theorem

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2005-729 J. Nešetřil and I. Švejdarová
Diameters of duals are linear

We prove that dual graphs and relational structures are connected. Moreover, these exponential structures have a linear diameter, which we determine up to a constant.

2005-730 D. Piguet and M. Stein
The Loebl Conjecture for trees of small diameter

The Loebl Conjecture asks whether any graph on n vertices, at least half of which have degree $\frac{n}{2}$, contains any tree of order $\frac{n}{2}+1$ as a subgraph. We prove the conjecture for trees with diameter ≤ 5.

2005-731 M. Ghebleh, D. Král', S. Norine, and R. Thomas
The circular chromatic index of flower snarks

We determine the circular chromatic index of flower snarks, by showing that $\chi'_c(F_3) = 7/2$, $\chi'_c(F_5) = 17/5$ and $\chi'_c(F_k) = 10/3$ for every odd integer $k \geq 7$, where F_k denotes the flower snark on $4k$ vertices.

2005-732 J. Nešetřil and P. Ossona de Mendez
The Grad of a Graph and Classes with Bounded Expansion

We introduce classes of graphs with *bounded expansion* as a generalization of both proper minor closed classes and degree bounded classes. Such classes are based on a new invariant, the *greatest reduced average density (grad) of* G *with rank* r, $\nabla_r(G)$. We generalize to these classes some results proved for proper minor closed classes and bounded degree graphs, such as the existence of low tree-width colorings and homomorphism dualities.

2005-733 Z. Dvořák, ed.
XI. Midsummer Combinatorial Workshop

2005-734 Z. Dvořák, R. Škrekovski, and M. Tancer
List-Colouring Squares of Sparse Subcubic Graphs

The problem of colouring the square of a graph naturally arises in connection with the distance labelings, which have been studied intensively. We consider this problem for sparse subcubic graphs and show that the choosability $\chi_\ell(G^2)$ of the square of a subcubic graph G of maximum average degree d is at most four if $d < 24/11$ and G does not contain a 5-cycle, $\chi_\ell(G^2)$ is at most five if $d < 7/3$ and at most six if $d < 5/2$. Wegner's conjecture claims that the chromatic number of the square of a subcubic planar graph is at most seven. Our result implies that $\chi_\ell(G^2)$ is at most four if $g \geq 24$, it is at most 5 if $g \geq 14$, and it is at most 6 if $g \geq 10$. For lower bounds, we find a planar subcubic graph G_1 of girth 9 such that $\chi(G_1^2) = 5$ and a planar subcubic graph G_2 of girth five such that $\chi(G_2^2) = 6$. As a consequence, we show that the problem of 4-colouring of the square of a subcubic planar graph of girth $g = 9$ is NP-complete. We conclude the paper by posing few conjectures.

2005-735 R. N. Ball, A. Pultr, and P. Vojtěchovský
Colored Graphs without Colorful Cycles

A colored graph is a complete graph in which a color has been assigned to each edge, and a colorful cycle is a cycle in which each edge has a different color. We first show that a colored graph lacks colorful cycles iff it is Gallai, i.e., lacks colorful triangles. We then show that, under the operation $m \circ n \equiv m + n - 2$, the lengths of omitted cycles in a colored graph form a monoid isomorphic to a submonoid of the natural numbers which contains all integers past some point. We conjecture that all such monoids are realized, and prove that several are.

We then characterize exactly Gallai graphs, i.e., graphs in which every triangle has edges of exactly two colors. We show that these are precisely the graphs which can be iteratively built up from three simple colored graphs, having 2, 4, and 5 elements, respectively. We then characterize, in two quite different ways, the monochromes, i.e., the connected components of maximal monochromatic subgraphs, of exact Gallai graphs. The first characterization is in terms of their reduced form, a notion which hinges on the important idea of a full homomorphism. And the second characterization is by means of a homomorphism duality

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2005-736 J. Kára, J. Kratochvíl, and D. R. Wood
On the Complexity of the Balanced Vertex Ordering Problem

We consider the problem of finding a balanced ordering of the vertices of a graph. More precisely, we want to minimise the sum, taken over all vertices v, of the difference between the number of neighbours to the left and right of v. This problem, which has applications in graph drawing, was recently introduced by Biedl *et al.* . They proved that the problem is solvable in polynomial time for graphs with maximum degree three, but $\mathcal{NP}$-hard for graphs with maximum degree six. One of our main results is closing the gap in these results, by proving $\mathcal{NP}$-hardness for graphs with maximum degree four. Furthermore, we prove that the problem remains $\mathcal{NP}$-hard for planar graphs with maximum degree six and for 5-regular graphs. On the other hand we present a polynomial time algorithm that determines whether there is a vertex ordering with total imbalance smaller than a fixed constant, and a polynomial time algorithm that determines whether a given multigraph with even degrees has an 'almost balanced' ordering.

2005-737 J. Fiala and J. Kratochvíl
On the computational complexity of the $L_{(2,1)}$-labeling problem for regular graphs

An $L_{(2,1)}$-labeling of a graph of span t is an assignment of integer labels from $\{0, 1, \ldots, t\}$ to its vertices such that the labels of adjacent vertices differ by at least two, while vertices at distance two are assigned distinct labels.

We show that for all $k \geq 3$, the decision problem whether a k-regular graph admits an $L_{(2,1)}$-labeling of span $k+2$ is NP-complete. This answers an open problem of R. Laskar.

2005-738 J. Fiala, J. Kratochvíl, and A. Pór

On the computational complexity of partial covers of Theta graphs

By use of elementary geometric arguments we prove the existence of a special integral solution of a certain system of linear equations. The existence of such a solution then yields the NP-hardness of the decision problem on the existence of locally injective homomorphisms to Theta graphs with three distinct odd path lengths.

2005-739 J. Nešetřil and P. Ossona de Mendez

Grad and Classes with Bounded Expansion I. Decompositions.

We introduce classes of graphs with *bounded expansion* as a generalization of both proper minor closed classes and degree bounded classes. Such classes are based on a new invariant, the *greatest reduced average density (grad) of G with rank r*, $\nabla_r(G)$. For these classes we prove the existence of several partition results such as the existence of low tree-width and low tree-depth colorings. This generalizes and simplifies several earlier results (obtained for minor closed classes).

2005-740 J. Nešetřil and P. Ossona de Mendez

Grad and Classes with Bounded Expansion II. Algorithmic Aspects.

Classes of graphs with *bounded expansion* are a generalization of both proper minor closed classes and degree bounded classes. Such classes are based on a new invariant, the *greatest reduced average density (grad) of G with rank r*, $\nabla_r(G)$. These classes are also characterized by the existence of several partition results such as the existence of low tree-width and low tree-depth colorings. These results lead to several new linear time algorithms, such as an algorithm for counting all the isomorphs of a fixed graph in an input graph or an algorithm for checking whether there exists a subset of vertices of *a priori* bounded size such that the subgraph induced by this subset satisfies some arbirtrary but fixed first order sentence. We also show that for fixed p, computing the distances between two vertices up to distance p may be performed in constant time per query after a linear time preprocessing. We also show, extending several earlier results, that a class of graphs has sublinear separators if it has sub-exponential expansion. This result result is best possible in general.

2005-741 J. Nešetřil and P. Ossona de Mendez

Grad and Classes with Bounded Expansion III. Restricted Dualities

We study restricted homomorphism dualities in the context of classes with bounded expansion. This presents a generalization of restricted dualities obtained earlier for bounded degree graphs and also for proper minor closed classes.

This is related to distance coloring of graphs and to the "approximative version" of Hadwiger conjecture.

2005-742 J. Nešetřil
Strom jako matematická struktura – i v umění

Trees as mathematical structures and its significance in math as well as in art. *(in Czech)*

2005-743 M. Bodirsky and J. Kára
The Complexity of Equality Constraint Languages

We apply the algebraic approach to infinite-valued constraint satisfaction to classify the computational complexity of all constraint languages where the constraint types are Boolean combinations of the equality relation. We show that such a constraint language is tractable if it admits a constant unary or an injective binary polymorphism, and is NP-complete otherwise.

2005-744 R. Naserasr, Y. Nigussie and R. Škrekovski
Homomorphisms of triangle-free graphs without a K_5-minor

In the course of extending Grötzsch's theorem, we prove that every triangle-free graph without a K_5-minor is 3-colorable. It has been recently proved that every triangle-free planar graph admits a homomorphism to the Clebsch graph. We also extend this result to the class of triangle-free graphs without a K_5-minor. This is related to some conjectures which generalize the Four-Color Theorem. While we show that our results cannot be extended directly, we conjecture that every K_6-minor-free graph of girth at least 5 is 3-colorable.

2005-745 E. Ondráčková and T. Valla (eds.)
Spring School on Combinatorics 2005

2005-746 J. Černý
A simple proof for open cups and caps

Let X be a set of points in general position in the plane. General position means that no three points lie on a line and no two points have the same x-coordinate. $Y \subseteq X$ is a *cup*, resp. *cap*, if the points of Y lie on the graph of a convex, resp. concave function. Denote the points of Y by $p_1, p_2, \ldots, p_m$ according to the increasing x-coordinate. The set Y is *open* in X if there is no point of X above the polygonal line $p_1, p_2, \ldots, p_m$. Valtr showed that for every positive integers k and l there exists a positive integer $g(k, l)$ such that any $g(k, l)$-point set in the plane in general position contains an open k-cup or an open l-cap. This is a generalization of the Erdős-Szekeres theorem on cups and caps. We show a simple proof for this theorem and we also show better recurreces for $g(k, l)$. This theorem implies results on empty polygons in

k'-convex sets proved by Károlyi et. al., Kun and Lippner and Valtr.A set of points is k'-convex if it determines no triangle with more than k' points inside.

2005-747 J. Kára, ed.
Workshop on Graph Classes, Width Parameters and Optimization 2005

2005-748 T. Kaiser, D. Král', and L. Stacho
Tough spiders

Spider graphs are the intersection graphs of subtrees of subdivisions of stars. Thus, spider graphs are chordal graphs that form a common superclass of interval and split graphs. Motivated by previous results on the existence of Hamilton cycles in interval, split and chordal graphs, we show that every 3/2-tough spider graph is hamiltonian. The obtained bound is best possible since there are $(3/2-\varepsilon)$-tough spider graphs that do not contain a Hamilton cycle.

2005-749 M. Bálek, D. Hartman, and J. Kára, eds.
Open Problems for Homonolo 2005

2005-750 M. Nehéz and D. Olejár
On Dominating Cliques in Random Graphs

Motivated by the communication problems in large-scale networks, we study the dominating cliques in random graphs in this paper. Our main result points out conditions for an existence of dominating cliques in random graphs $\mathbb{G}(n,p)$ in the terms of bounds on the probability p.

2005-751 D. M. Jackson, I. Moffatt, and A.H. Morales
On the group-like behaviour of the Le-Murakami-Ohtsuki invariant

We study the effect of Feynman integration and diagrammatic differential operators on the structure of group-like elements in the algebra generated by coloured vertex-oriented uni-trivalent graphs. We provide applications of our results to the study of the LMO invariant, a quantum invariant of manifolds. We also indicate further situations in which our results apply and may prove useful. The enumerative approach that we adopt has a clarity that has enabled us to perceive a number of generalizations.

2005-752 P. Ossona de Mendez and P. Rosenstiehl
Encoding pointed maps by double occurrence words

We show that pointed maps with m edges are in bijection with standard double occurrence words with $(m+1)$ symbols.

2005-753 A. Pór and D. R. Wood

Colourings of the Cartesian Product of Graphs and Multiplicative Sidon Sets

Let $\mathcal{F}$ be a family of connected bipartite graphs, each with at least three vertices. A proper vertex colouring of a graph G with no bichromatic subgraph in $\mathcal{F}$ is *$\mathcal{F}$-free*. The *$\mathcal{F}$-free chromatic number* $\chi(G, \mathcal{F})$ of a graph G is the minimum number of colours in an $\mathcal{F}$-free colouring of G. For appropriate choices of $\mathcal{F}$, several well-known types of colourings fit into this framework, including acyclic colourings, star colourings, and distance-2 colourings. This paper studies $\mathcal{F}$-free colourings of the cartesian product of graphs.

Let H be the cartesian product of the graphs $G_1, G_2, \ldots, G_d$. Our main result establishes an upper bound on the $\mathcal{F}$-free chromatic number of H in terms of the maximum $\mathcal{F}$-free chromatic number of the G_i and the following number-theoretic concept. A set S of natural numbers is *k-multiplicative Sidon* if $ax = by$ implies $a = b$ and $x = y$ whenever $x, y \in S$ and $1 \leq a, b \leq k$. Suppose that $\chi(G_i, \mathcal{F}) \leq k$ and S is a k-multiplicative Sidon set of cardinality d. We prove that $\chi(H, \mathcal{F}) \leq 1+2k \cdot \max S$. We then prove that the maximum density of a k-multiplicative Sidon set is $\Theta(1/\log k)$. It follows that $\chi(H, \mathcal{F}) \leq \mathcal{O}(dk \log k)$. We illustrate the method with numerous examples, some of which generalise or improve upon existing results in the literature.

2005-754 P. Bella, D. Král', B. Mohar, and K. Quittnerová

Labeling planar graphs with a condition at distance two

An $L(2, 1)$-labeling of a graph is a mapping $c : V(G) \to \{0, \ldots, K\}$ such that the labels assigned to neighboring vertices differ by at least 2 and the labels of vertices at distance two are different. The smallest K for which an $L(2, 1)$-labeling of a graph G exists is denoted by $\lambda_{2,1}(G)$. Griggs and Yeh [SIAM J. Discrete Math. 5 (1992), 586–595] conjectured that $\lambda_{2,1}(G) \leq \Delta^2$ for every graph G with maximum degree Δ. We prove the conjecture for planar graphs with maximum degree $\Delta \neq 3$. All our results also generalize to the list-coloring setting.

2005-755 M. Loebl

Lecture Notes on Matroids

These are lecture notes for the first part of the lecture "Introduction to Mathematical Programming".

2005-756 F. Mráz

On the Maximal Set of Feasible Coefficients in Interval Linear Systems

The paper deals with a system of linear equations Ax = b whose input data are described by intervals. By a set of feasible coefficients we mean a set F containing all matrices A and vectors b in given intervals for which the corresponding system Ax=b has a nonnegative solution.In this way, the set F is closely connected with an interval linear programming problem. A description of the set F is given which enables to construct a maximal set of feasible coefficients.

2000 Mathematics Subject Classification. 65G40, 90C05.

Key words and phrases. Interval coefficients, systems of linear equations, nonnegative solutions, linear programming.

2005-757 L. Clark

Asymptotic Distribution of the Sum of the Lengths of Ascents or of Descents in Permutations

Let $A_n(\sigma)$ denote the sum of the lengths of ascents of a permutation σ of $\{1, \ldots, n\}$ chosen randomly according to a uniform distribution. We find the exact expectation and variance of A_n and then show that the A_n are asymptotically normally distributed. An identical result holds for the sum of the lengths of descents of a permutation.

Key Words: Ascents, Descents, Asymptotic Normality
