GRAPH THEORY DAY 2

Nečtiny

8. 10. 1999

(edited by Roman Čada and Jana Maxová)

After success of DIMATIA Day in September 1999 in Prague we decided to organize the second DIMATIA Day in Nečtiny. We organized it jointly with West-Bohemia University in Plzeň /Pilsen/ who is owning the Nečtiny Castle Conference Centre.

This was a very fruitful and yet informal meeting attended by 21 mathematicians from Czech Republic and we were pleased that our friends Roman Nedela, Stanislav Jendrol' and Robert Jajcay from Slovakia took part. Also Moshe Rosenfeld /presently Fullbright fellow at DIMATIA in Prague/ took part and thus the meeting was held in English.

This was the last Graph Theory Meeting which was attended by Dr. Ivan Havel, CSc who unexpectedly died in December 99. He will be remembered by his colleagues and students and by his scientific work.

To his honour we decided to include some of his beautiful problems which he posed.

I thank to Zdeněk Ryjáček and to his collaborators in Pilsen for organizing this meeting. The meeting was supported by DIMATIA GAČR grant 0242/99.

Jaroslav Nešetřil

Dedicated to the memory of Ivan Havel

List of participants

Roman Čada
Ivana Čadová
Dalibor Fronček
Ivan Havel
Jiří Holenda
Přemysl Holub
Robert Jajcay
Stano Jendrol'
Tomáš Kaiser
Martin Klazar
Martin Loebl
Ladislav Nebeský
Roman Nedela
Jarik Nešetřil
Helena Nešetřilová
Moshe Rosenfeld
Zdeněk Ryjáček
Marta Ryjáčková
Libuše Tesková
Milena Tuchanová
Pavel Valtr
Bohdan Zelinka

The Castle at Nečtiny

This meeting took place at the castle at Nečtiny which is located about 35 km north from Pilsen, near to the small town Manětín.

The castle was built by Václav Gryspek of Gryspach under the walls of an older and destroyed castle named Preitenštejn in second half of 16th century. The original name of the castle Preitenštein was Breitenstein and it was built by Oldřich Pluh of Rabštejn in the first half of 14th century. But in 1623 the castle was recovered by this family for a participation in an uprising of czech aristocracy.

The next important owners in the history of the castle were Kokořovci of Kokořov, who added to the building a new wing in Baroque style.

From 1839 till 1945 the domain of Nečtiny was owned by the family Mensdorf-Pouilly. Earl Alfons Mensdorf-Pouilly made the castle his main residency. During the years 1855–1858 the castle was rebuilt in New-Gothic style. So the New-Gothic chapel in the floor and two new stylish towers ensued. In 1858 a sepulchre of Mensdorf's family in New-Romanesque style was built up.

In 1964 the castle burned down. It was renovated in 1970 for the purpose of school of agriculture. University of West Bohemia is the owner of the castle from 1998.

Programme

11.00 - 11.45	Moshe Rosenfeld
11.45 - 12.30	*Problem session*
lunch	
14.30 - 15.15	Stano Jendrol'
15.15 - 15.45	Jarik Nešetřil
15.45 - 16.15	Roman Nedela
coffee break	
16.45 - 17.15	Ladislav Nebeský
17.15 - 17.45	Martin Loebl

Some favourite problems of Ivan Havel

1. **Coloring of planar graphs**
   Is it true that there exists a positive k with the following property: if G is a planar graph where any two triangles have distance $\geq k$ then $\chi(G) \leq 3$?
   ($k \geq 2$ was proved by I. Havel disproving a conjecture of Grünbaum)

2. **Hamiltonian graphs**
   Is it true that for every odd n the middle level subgraph of n-dimensional cube is Hamiltonian ?
   (a middle level graph is induced by vertices of the cube which contain $\lfloor n/2 \rfloor$ or $\lceil n/2 \rceil$ zeros.)

3. **Trees in cubes**
   Every binary tree on 2^n vertices which is balanced (i.e. it can be colored by two colors such that both color classes are of the same size) can be embedded into the n-dimensional cube.

Abstracts of talks at Graph Theory Day 2

EQUIANGULAR LINES AND GRAPHS

Moshe Rosenfeld

A set of lines through the origin in R^d is called equiangular if the angle among any pair of distinct lines is the same. There is a 1-1 correspondence between sets of n $(> d)$ equiangular lines in R^d and switching classes of graphs. In particular, an immediate consequence of this is that there are only finitely many possible angles for which there are $d+1$ equiangular lines in R^d.

In this talk I'll survey the background and discuss some on-going research as well as open problems.

ON TYPES OF FACES IN PLANE GRAPHS

Stanislav Jendrol'

Let $G = (V, E, F)$ be a connected plane graph with vertex set V, edge set E and face set F. A face f is an $\langle a_1, a_2, \ldots, a_m \rangle$-face if f is an m-gon and the degrees $d(x_i)$ of the vertices $x_1, \ldots, x_m$ incident with f in the cyclic order are $a_1, \ldots, a_m$, respectively. The lexicographic minimum $\langle b_1, \ldots, b_m \rangle$ such that f is a $\langle b_1, ..., b_m \rangle$-face is called the type of f. We give a survey on results concerning face types in polyhedral graphs.

GOOD CHARACTERIZATIONS FOR COLORING PROBLEMS

Jaroslav Nešetřil

Many coloring problems may be expressed by means of the existence of homomorphisms into a particular graph H. In this context we speak about H-coloring problem. With Claude Tardif we recently characterized those coloring problems which can be characterized by means of a finite set of forbidden subgraphs. We outlined the proof of this result.

CHIRAL MAPS AND HYPERMAPS

Roman Nedela

Regular hypermap on an orientable surface is called chiral if it is not isomorphic to its mirror image. We study the chirality of maps and hypermaps in detail. We show that it is possible to measure the chirality of hypermaps in terms of a quotient of two groups associated with any hypermap. In this way we introduce a new invariant - chirality index of a hypermap. We derive some general results related to chirality index. Futhermore we focus our attention to classification of all chiral hypermaps on surfaces with genus up to four. We demonstrate construction techniques on examples.

(Most of the presented results were done in collaboration with Antonio Breda d'Azevedo and Martin Skoviera.)

CHARACTERIZING GEODETIC GRAPHS

Ladislav Nebeský

Two characterizations of geodetic graphs will be presented. The first characterization is based on the notion of the interval function (in the sense of H. M. Mulder). The second one utilizes a certain binary operation defined on the vertex set of a connected graph.

GRAPH THEORY OF CRYSTAL STRUCTURES

MARTIN LOEBL

A lattice graph or a 'regular' finite subgraph of a lattice graph in a finite dimension d is sometimes called a crystal structure. A graph theory of crystal structures studies classical graphs problems (on crystal structures), but the methods and the results often differ from the study of general graphs. In my lecture, after a short general introduction, I present recent results on enumeration of perfect matchings of a three dimensional cube (sometimes called the 3-dimensional dimer problem) and on enumeration of the edge-cuts of a three dimensional cube (called the 3-dimensional Ising problem).

Problems presented at Graph Theory Day 2

Jarik Nešetřil

Problem 1 *(UNIVERSAL GRAPHS FOR PLANAR GRAPHS)* Is it true that there exists a triangle-free (not necessarily planar) graph H such that any triangle free planar graph G is homomorphic to H (i.e. there exists a mapping $f : V(G) \to V(H)$ such that $\{f(x), f(y)\} \in E(H)$ for any edge $\{x, y\} \in E(G)$). (In this setting, any triangle free planar graph G is homomorphic to triangle K_3.)

Tomáš Kaiser asked whether there exists K_4-free graph H such that any planar graph is homomorphic to H.

Let us remark that it is not clear how to prove (without use 4 Color Theorem) that there exists a K_5-free graph H such that any planar graph is homomorphic to H.

(see J. Nešetřil: Aspects of Structural Combinatorics, KAM DIMATIA Series 99-435; to appear in Taiwanese J. Math.)

Problem 2 *(COMPLEXITY OF SWITCHING PROPERTIES)* We say that graphs $G = (V, E)$, $G' = (V, E')$ are *switching equivalent* ($G \sim G'$) if the symmetric difference $E \,\Delta\, E'$ is a complete bipartite graph. (This definition is well known and due to J. Seidel.) We say that G is F-free if G does not contain an induced subgraph isomorphic to F.

The following complexity problem has been investigated by [KNZ]:

F-free switching
Input: a graph G
Question: Does there exists a graph G', $G' \sim G$ such that G' is F-free.

Somewhat surprisingly this problem turns to be polynomially decidable in all cases which were considered. However this is still an open problem for a general graph F.

[KNZ] J. Kratochvíl, J. Nešetřil, O. Zyka: On the computational complexity of Seidel's switching. In: Ann. Discrete Math. 51, North Holland, Amsterdam 1992, 161–166

Martin Klazar

Problem A (not necessarily simple) graph $G = (\{1, 2, \ldots, m\}, E)$ is *non-crossing* (NC) if it does not have two edges $e_1 = \{x_1, y_1\}$ and $e_2 = \{x_2, y_2\}$ such that $x_1 < x_2 < y_1 < y_2$. Let a_n be the number of NC graphs which have n vertices, where isolated vertices are allowed but multiple edges are forbidden. Let b_n be the number of NC graphs which have n edges, where isolated vertices are forbidden but multiple edges are allowed. Using generating functions, one can prove that for $n \geq 3$

$$a_n = 8b_{n-2}.$$

Give a bijective proof of this identity.

Zdeněk Ryjáček

Let G be a claw-free graph. The graph obtained from G by replacing the neighborhood of a vertex x with connected noncomplete neighborhood by a clique is called the local completion of G at x. The (uniquely determined) graph obtained from a claw-free graph G by recursively performing the local completion operation as long as this is possible is called the *(claw-free) closure of G*. The closure is a line graph (of a triangle-free graph) and has the same circumference as G. The following result is known.

Theorem (S. Brandt, O. Favaron, Z. R.)
For every $k \geq 2$ there is a k-connected nonpancyclic claw-free graph with pancyclic closure.

On the other hand, it is known that if a claw-free graph G has complete (and therefore pancyclic) closure, then G contains cycles C_3, C_4, C_5 (with one exception), C_{n-1}, C_n.

Conjecture Let $c \geq 0$. Then for large n, every claw-free graph G of order n whose closure is complete contains cycles C_i for all i, where $3 \leq i \leq 3 + c$ and $n - c \leq i \leq n$.

Problem Determine the maximum number of cycle lengths that can be missing in a claw-free graph with complete closure.

Stanislav Jendrol'

Problem 1 It is easy to prove that any two longest paths (cycles) in a connected (2-connected, respectively) graph have a vertex in common. We believe that the following is true:

Conjecture. *Every three longest paths (cycles) in a connected (2-connected, respectively) graph have a vertex in common.*

This conjecture is close to a problem of Zamfirescu which is formulated in Voss [3]: Determine p (resp. c) which is the largest integer k such that any k longest paths (k longest cycles) of a connected (2-connected) graph have a vertex in common. It is known that $p \leq 6$ for both, paths and cycles, see [1, 2].

[1] S. Jendrol' and Z. Skupień, Exact numbers of longest cycles with empty intersection. Europ. J. Combinatorics 18 (1997), 575-578.

[2] Z. Skupień: Smallest sets of longest paths with empty intersection. Combinatorics, Probability and Computing 5 (1996), 429-436.

[3] H.-J. Voss, Cycles and Bridges in Graphs, Kluwer Academic, Dordrecht, 1992.

Problem 2 *(S. J. and Z. Skupień)* Let G be a cubic 3-connected bipartite planar graph whose edge set can be decomposed into three subsets E_1, E_2, E_3 such that the subgraphs induced by the edges of E_1 and E_2 are trees, and the subgraph induced by the edges of E_3 is a cycle. We believe that the following is true:

Conjecture: *G is hamiltonian.*

Note that if E_2 contains at most one edge then G is hamiltonian, see [1].

[1] S. Jendrol' and Z. Skupień, W_v Cycles in plane graphs. Geometriae Dedicata 55 (1995), 293-303.