- Zykrov problem
- on algorithmic solvability of Ternarycat

MATHMATICAL LINGUISTICS
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KAM Series
Nevertheless the solution is known only for special classes of $L$-Z problem has been studied by many authors.

Introduction

Peter Brusek, Kostelec

On Algorithmic Solvability of Trachtenberg-Zykov Problem
The post correspondence system is an ordered pair 
\[ \langle \alpha, \beta \rangle \]
where \( \alpha, \beta \) are finite strings of non-empty finite words over 

**Example** (3): 

which is also known to be uncomputable and unsolvable. (see [1972].) We use the classification of "Post correspondence problem" (Kahr, Moore and Hao, Wang's unsolvability of "domino problem") within the algorithmic 

Buttiko proved theorem A using the algorithmic 

I.2. Post correspondence systems

... problem is studied and a new proof of theorem A is given.

In the second part, the infinite modulations of Z

... there exists a finite graph \( G \) whose neighborhood set is \( N \).

Thereom B: There exists no algorithm which, given a

paper we prove:

... problem is unknown till now. In the first part of the present

An analogous result for the infinite modulation of Z

\[ \{H\} = (G) \]

Finite graph \( H \), will determine whether there exists a graph \( G \)

Thereom A: There exists no algorithm which, given a

Buttiko proved in [I] that Z-problem is algorithmically unsolvable in the case

star (for a survey see [2]). Thus fact led to the conjecture

of graphs such as paths, cycles, graphs homeomorphic to a
First we will study the relationship between post-modification of T-Z problem and the existence of labeled graphs with a prescribed neighborhood set.

II. On the algorithmic solvability of the finite system.

Solve Post problem in finitely many steps for any given Post system. There is no algorithm which can determine whether S has a solution. The Post correspondence problem for S is to determine whether

\[ \text{Example I: Let } S = \{A, B\}, \quad A \Rightarrow [u_1 u_2], \quad B \Rightarrow [v_1 v_2]. \]

and \( A, B \).

The symbol \( v \) denotes the concatenation of the words \( u \).

\[ u_1 u_2 \cdots u_n = v_1 v_2 \cdots v_m \]

from the set \( \{1, 2, \ldots, n\} \) such that

A solution of \( S \) is a finite sequence of numbers

\[ [u_1, v_1] = [u_2, v_2] = \cdots = [u_n, v_n] \cdot \{a, b\}. \]
II.2 Construction of the graph \((G_1, f_1)\), that \(N_G(\emptyset) \not\supseteq (S, f, \phi)\) then \(S\) has a solution.

2. If there exists a finite labeled graph \((G, f)\) such that \(N_G(\emptyset) \not\supseteq (S, f, \phi)\) then \(S|_1\) has a solution, then \(N_G(\emptyset) \not\supseteq (S, f, \phi)\).

Following properties to assist in any post system is a system of sets \((W, G)\) with the post system is having a solution and we describe a method now for every set of the finite labeled graph \((G, f, \phi)\) to every other.

Understand the set \(\{(g) \in \emptyset\} \cup \emptyset\) by \(f\)-neighborhood set of a labeled graph \((G, f)\) to \(U\). By \(f\)-neighborhood set of a labeled graph \((G, f)\), the \(f\)-neighborhood of \(g\) is defined as the rooted labeled graph \((G, f, \phi)\) is called the root of the vertex \(g\)

The \(\emptyset\)-neighborhood of \(g\) is the vertex \(f\). Then \(f\) is called the root of the vertex \(g\).

The labeled graph is an ordered pair \((G, f)\), where \(G\) is an

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Finally, we add co-colored vertices 

\[ (x^t + 1, z^t), (x^t, z^t + 1), (x^t, z^t), (x^t, z^t - 1) \]

and edges

\[ (x^t + 1, z^t), (x^t, z^t + 1), (x^t, z^t), (x^t, z^t - 1) \]

for every \( t \geq 1 \). The resulting graph \( S \) is

\[ E(S) \]

belong to \( E(G) \). For \( k = 1, 2, \ldots, t - 1 \), let

\[ (x^k, y^k) \quad \text{and} \quad (y^k, z^k) \]

be two colored and let the next vertex \( z^k \) be added to \( S \), where \( z^k \) is the first character of \( y^k \) and \( z^k \) is the first character of \( x^k \). Similarly, for every \( k \) such that \( a_k \) is the last character of \( x^k \) and \( b_k \) is the last character of \( y^k \). Furthermore, for every pair of the vertices \( x^k, y^k \),

If \( q \in (x^k, y^k) \), then \( q = (x^k, y^k) \), otherwise \( q = (x^k, y^k) \).

For every \( i = 1, 2, \ldots, t \), let

\[ (x^i, y^i) \quad \text{and} \quad (y^i, z^i) \]

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Now we describe the construction of a labeled graph.

\[ (x^t, y^t, \ldots, z^t) \]

where \( (x^t, y^t, \ldots, z^t) = (S) \).

The word \( w \) can be divided into a sequence of co-colored vertices. And in this sense \( g \) is said to be a character of \( x \).
Next we give the exact description of \( \mathcal{M}(S) \) and \( \mathcal{W}(S) \).

Connect a vertex adjacent to another one with edge. The empty symbol \( \mathcal{V} \) written at a color, is written on the vertex of every vertex the set of all graphs from \( \mathcal{M}(S) \) and \( \mathcal{W}(S) \). Let the set of all graphs from the set \( \mathcal{M}(S) \), \( \mathcal{W}(S) \), and \( \mathcal{V} \) be a post system and let 4, \ldots, \( g \) successively.

A 4-colored root and the next \( m \) vertices are labeled by 4-colored root, whose universal vertex is the wheel with \( m+1 \) vertices, whose universal vertex is the \( \mathcal{M}(S) \) denotes the graph labeled graph isomorphic to the symbol \( \mathcal{V} \). Let 4, \ldots, \( g \) denote a vertex adjacent to all.

By a wheel, we mean a graph obtained from a cycle by adding one universal vertex, i.e., a vertex adjacent to all.
Now we define \( M(S) \) as the system of all subsets of \( \{ (v, j) \in S, t, v \in T \} \) where \( t = \sum_{w \in T} 4^w \cdot e \). The set \( M(S) \) contains all graphs with \( S(t) \). Let \( t' \) substitute elements of \( t \).
Claim 2 and defininition of $W(G)$ imply that if $\bar{f}(x) = f(y)$ then $f(z) = f(x)$. From the definition of the vertices of $W(G)$ we obtain

Moreover, from the definition of $W(G)$ and from the form of $f(e)$ as $f(e) = f(x)^{e(x)} f(y)^{e(y)} f(z)^{e(z)}$, necessary for $\bar{f}(x) = f(y)$, we have

Since there is no vertex in $G$ with the property of $\bar{f}(x)$, each must contain a vertex adjacent to $x$, different from $x$ and different from $x$. Let $e$ be adjacent to $x$, and adjacent to $x$. Then $e$ is such that $\bar{f}(x) = f(y)$, and the $e$-colored vertices $\bar{f}(x)$, $f(y)$, and $f(z)$ of $G$ satisfy the following conditions:

1. In the first case the $e$-neighborhood of $x$ contains the $e$-neighborhood of $f(y)$
2. In the second case we can repeat a similar argument with the next uncolored vertex adjacent to $x$ and different from $x$.

The cycle $x, y, z, x, y, z, x, y, z,$ etc.

Claim 2: $a = t$,
Consider the available e-colored neighborhoods of $t$, $\mathcal{N}(t)$, and the edges $(x,t)$. Then $t$ is adjacent to at least one vertex in each neighborhood of $t$, $\mathcal{N}(t)$. Let $a \in \mathcal{N}(t)$ and $b \in \mathcal{N}(t)$. By Theorem 2.5, the e-colored neighborhood of $a$ must contain at least one vertex in $\mathcal{N}(t)$.

Claim 1: $G$ contains the cycle $z^2_{x'} z^2_{x'} z^2_{x'} z^2_{x'} z^2_{x'} z^2_{x'} z^2_{x'} z^2_{x'}$.

Observe that $G$ is colored and we number the vertices $v_0, v_1, \ldots, v_9$ in such a way that for each $z^2_{x'}$, there exist vertices $v_i$ and $v_{i+1}$ (with the notation $v_i$) such that there exist two vertices $v_i$ and $v_{i+1}$ in $G$ with the following properties: $v_i$ and $v_{i+1}$ are adjacent to $x$.

This implies that if $p$ can be divided into vertex disjoint paths, $v_0, \ldots, v_{8}$, then the $e$-co-colored path $v_0, v_1, \ldots, v_{8}$ is isomorphic to a path.

If $x \notin \mathcal{N}(t)$, then the restriction of the $e$-co-colored neighborhood of $x$ to the set of all vertices colored by $c'z^2_{x'}$ adjacent to $x$ is isomorphic to a path from $c'z^2_{x'}$.

Let $x \in \mathcal{N}(t)$.

If $x \notin \mathcal{N}(t)$, then there exists an uncolored vertex next to $x$.

Corollary: If $G$ is an e-colored graph, then $G$ contains a $z^2$-co-colored vertex. (The existence follows from the above.)
Suppose that the vertex sets of graphs $G_n$ for $n \geq 1$ are
are by a complete graph with $\binom{n+1}{2}$ vertices.

mapping $\phi$ is not the root of $I$, then we denote

Let $I$ be a finite labeled

be a finite labeled

integer greater than 1. Let $I$ be a finite labeled

we denote by $\text{deg}(x)$ the degree of a vertex $x$ in $G_n$.

Before doing that we introduce some useful deinitions.

Type I equivalent.

In this part we show that the existence of labeled and

unlabeled neighborhoods with preseeted neighborhoods of a special

II.5. Labeled and unlabeled graphs with preseeted neighborhoods

written successively create the word

(1) is the solution of the Post system $S$ and the

$1, z, \ldots, z, 1, \ldots, 1$
then there exists a (finite) labeled graph \( G, f \) such that

\[ \phi = (G, f, \gamma) \]

If there exists a (finite) labeled graph \( G \) such that \( N(G) = M \)

\[ \leq \]

Lemma 3: Let \( G \) be a post system, \( M \in (\gamma) \), and \( \phi \) be a one-to-one mapping from \( \gamma \) to \( \in \). It is said to be good, if \( G \) is a one-to-one mapping with a partitionable property. The converse implication holds for the case of the

\[ \phi = (G, f, \gamma) \]

It is not difficult to see that \( N(G) \).

For every \( \gamma \), \( \lambda \), \( \mu \) and \( N(G) \in (\gamma, \lambda, \mu) \) and \( E(G) \in (\lambda, \mu, \gamma) \). 

\[ \lambda \subseteq (\gamma, \lambda, \mu) \] 

The vertices for every \( \gamma \) (the set \( V(G) \) for every \( \gamma \)) are

\[ \phi \in (\lambda) \]

Proof: Let \( G \) be a complete graph with \( V(G) \) for every \( \gamma \). Define \( \phi \) and \( \lambda \).

If \( \phi = (G, f, \gamma) \), then there exists a (finite) labeled graph \( G, f \) such that \( N(G) = M \).

\[ \leq \]

Lemma 2: Let \( Z \) be a one-to-one mapping.

\[ \phi = (G, f, \gamma) \]

If \( M \) is a set of rooted labeled graphs (over \( \gamma \)), then

\[ \{ \} \]

Define a graph \( L \)
The validity of the definition follows from the following property of the factor graph $G$.

The factor graph $G$ is defined as follows:

1. The vertex $x$ is the equivalence class containing $x$.
2. For each vertex $y \in V(G)$, the equivalence class of $y$ is the partition of $V(G)$ into equivalence classes.

It is easy to see that $\equiv$ is the equivalence relation.

Proof: Let $N(G) = M$. Define a binary relation $\equiv$ for every $x' \in M$.
and from the above, the uniqueness of the solution depends on the set $\mathcal{M}$, and the sets $\mathcal{M}$.

Let $G$ be a finite graph with $N(G)$ such that $\mathcal{M}(G) = \mathcal{M}$. Then $G$ has a solution if and only if there exists a solution in $G$.

**Corollary:** Let $G$ be a finite graph and $\phi$ be an arbitrary vertex of $\mathcal{M}(G)$.

To prove that $\phi$ is a contradiction to the definition of $\mathcal{M}(G)$, we suppose that for some vertex $x \in \mathcal{M}(G)$, we have $\phi \neq x$. It is sufficient to prove that there exists another vertex $y \in \mathcal{M}(G)$ such that $\phi \neq y$.

Let $N(x)$ be the neighborhood of $x$, and let $N(y)$ be the neighborhood of $y$. Since $\phi \neq x$, we show that $\phi \neq y$ for all $y \in \mathcal{M}(G)$.

For some $g \in \mathcal{M}(G)$, we define the labeling $L(x)$ of $x$, and the labeling $L(y)$ of $y$ as follows:

$$(b)\phi = 1 + (1-(b)\phi) = ([x] \cup \mathcal{M}(G))$$

For every $x \in \mathcal{M}(G)$, we obtain a contradiction to the definition $L(x)$ is a wheel.
Given a Post system, solve the "infinite" Post correspondence problem for any problem can be proved that there is no algorithm which can determine whether \( S \) has an \( \omega \)-solution. Using a modification of the "infinite" Post correspondence problem for \( S \) is to then \( S \) has an \( \omega \)-solution, too.

These words, obviously, if a Post system \( S \) has a solution, where by \( \sum \), we mean the infinite concatenation of \( \sum \).

Let \( G = \{ S \} \) with \( S \in \{ 1, \ldots, n \} \) be a Post system, and

III. The "infinite" modification of the Post problem.

Problem and prove the known butterfly's result.

Modify all graph and the "infinite" modification of Post neighborhoods and the "infinite" modifications with preserved between the existence of infinite graphs with infinite Hamiltonian modification of Post problem. We will study the relationship between each way as in part II for the finite graph, a similar way as in part II for the finite graph.

III. On algorithmic solvability of the infinite

exists a finite graph \( G \) with \( N(G) \in \mathbb{N} \). Second set of finite graphs, will determine whether there

Theorem 1: There exists no algorithm which, given a
Graphs with prescribed neighborhood set.

III. The infinite modification of T-Z problem and

exists a graph \( G \) with \( N(G) = N \).

finite set \( N \) of finite graphs, will determine whether there

Corollary 3: There exists no algorithm which, given a

\{ G such that \( N(G) \epsilon W \), \( N \epsilon W \) \}

has an o-solution if and only if there exists a graph

Good.

Corollary 2: Let \( G \) be a post system and \( x \in Z^+ \) be

obtain

At lemma 2 and lemma 3 hold also for infinite graphs, we

\( \bigwedge_{i=1}^n \bigvee_{j=1}^m \bigwedge_{k=1}^l \)

succeedingly create the word written

coloured on the vertices of \( G \) (except the endvertex)

and find the infinite sequence \( \ldots t_1^i \).

such that the

of lemma 1, we can prove that \( F \) is a one-way infinite path

thus also an o-solution. Otherwise, similarly as in the proof

2-coloured vertex. If \( F \) is finite, then \( G \) has an o-solution

be some component of \( G \), \( \forall \epsilon (G) \), \( \forall \epsilon (G) \).

if \( G \) is a labelled graph with \( N(G) \epsilon W \)

only one 2-coloured vertex.

labelled graph \( G \) such that \( N(G) \epsilon W \).

in a similar way as in II. 2 we can assert to \( G \) the

Proof: (a) If a post system \( S \) has an o-solution, then

Labelled \( G \) such that \( N(G) \epsilon W \).

If there exists a labelled graph \( G \) such that \( N(G) \epsilon W \)
Then
\[ (\forall x \in \mathcal{V})(\exists y \in \mathcal{V})(x \sim y) \]

implies that \( \{x, y\} \) is an edge of \( (\mathcal{G}, \mathcal{E}) \).

This implies that \( \{x, y\} \) is an edge of \( (\mathcal{H}, \mathcal{E}) \).

For every \( \mathcal{G} \in (\mathcal{G}, \mathcal{E}) \) there exists the unique \( \mathcal{H} \) such that \( \mathcal{G} \subset \mathcal{H} \).

Let \( \mathcal{G} \) be an arbitrary vertex of \( \mathcal{G} \). We show that the

following holds:

Proof: Let \( \mathcal{G} \) satisfy the assumption of the Lemma, and

\[ \{\mathcal{H}_1, \ldots, \mathcal{H}_n\} = (\mathcal{G}) \]

Lemma 5: If there exists a (finite) graph \( \mathcal{G} \) such that

\[ \{\mathcal{H}_1, \ldots, \mathcal{H}_n\} = (\mathcal{G}) \]

let \( \mathcal{G} \) denote the disjoint union of graphs \( G_1, \ldots, G_n \),

and let \( \mathcal{G} \) be a countable graph with the same neighborhood set.

exists a countable vertex set, since for every infinite graph there

finite graphs. It is sufficient to continue to graphs with

We consider only finite neighborhood sets containing
The neighborhoods of the other vertices of $G$ are

If $j, f$ are not diastereomers, then $N(f \cup j, \Gamma)$ is the set of all $g \in \Gamma$ such that $g \equiv f \mod 3$ and $g \equiv j \mod 3$, for all $g \in \Gamma$. Let $\forall j \in \Gamma$ such that $\forall j \in \Gamma$. Now we define a graph $G$. Let $\forall j \in \Gamma$.

If $j \equiv 0 \mod 3$, then $G$ is isomorphic to $H_1 \sqcup H_2$.

If $j \equiv 2 \mod 3$, then $G$ is isomorphic to $H_1 \sqcup H_2$.

If $j \equiv 1 \mod 3$, then $G$ is isomorphic to $H_1 \sqcup H_2$.

Lemma 6: If there exists a graph $G$ with $G \equiv H \mod 3$, then there exists a graph $G$ with $G \equiv H \mod 3$. Suppose that the set $\{ \Gamma \}$ is infinite. If $ \Gamma \neq \emptyset$, then we can consider

Theorem (G) A $\exists x \in \Gamma$ such that $x \equiv H \mod 3$. Suppose that the set $\{ \Gamma \}$ is infinite.
Theorem 2: There exists no algorithm which, given a finite graph \( G \) with \( N(G) = \{v_1, \ldots, v_n\} \) and \( (G, F) \), determines whether the graph \( G \) is a complete graph.

According to Corollary 2, Lemma 5 and Lemma 6 we obtain:

\[
\{v_1, \ldots, v_n\} = (G, F) \quad \text{and} \quad \emptyset = (G, F)
\]

and only if there exists a graph \( G \) with \( N(G) = \{v_1, \ldots, v_n\} \) and \( (G, F) \) such that for every \( x \in V(G) \):

\[
\begin{align*}
\text{for } x \in V(G) & \quad \text{for every } x \in V(G) \\
(x, y) & \quad (x, y) \\
\end{align*}
\]

It is not difficult to see that \( N(x, G) \) is isomorphic to \( G \).

Let \( V(G) = \{v_1, \ldots, v_n\} \) and let \( E(G) \) contain the edges:

\[
\begin{align*}
\text{if } i \equiv 0 \pmod{2} & \quad \text{if } i \equiv 1 \pmod{2} \\
(1, i) & \quad (1, i) \\
\end{align*}
\]

and for every \( i \neq 1 \):

\[
\begin{align*}
\text{if } i \equiv 0 \pmod{2} & \quad \text{if } i \equiv 1 \pmod{2} \\
(i, j) & \quad (i, j) \\
\end{align*}
\]

Now let \( N(G) = \{v_1, \ldots, v_n\} \) and \( F \) be two bijections to \( V(G) \).

The lemma for \( n \geq 2 \) is insufficient to prove.

By a repeat of the procedure we obtain a graph \( G \) with \( N(G) = \{v_1, \ldots, v_n\} \) and thus \( N(G, F) \) isomorphic to \( (G, F) \).
References