KAMAK 2020

Penzion U Lucerny, Kytlice

September 20 – 25

Charles University, Prague

Organizers:

Pavel Dvořák
Robert Šámal

Brochure of open problems,
Prague, 2020.

Contents

Program

8:00 breakfast

9:00 morning session I

10:30 break

11:00 morning session II

12:30 lunch

15:00 afternoon session I

16:30 break

17:00 afternoon session II

18:30 progress reports

19:00 dinner

Wednesday is planned to be free to make an excursion in the neighborhood with everyone who would like to come.

OPEN PROBLEMS

Problem 1. *A function theoretic problem, with possible applications to digraphs (suggested by Carl Feghali)*

Statement and Motivation:

Consider two functions f and g from a set E into a set F such that $f(x) \neq g(x)$ for every $x \in E$. Suppose that there exists a positive integer n such that for any element z in F either

$$|f^{-1}(z)| \leq n \text{ or } |g^{-1}(z)| \leq n.$$

It was shown in [2] that E can be partitioned into $2n + 1$ subsets $E_1, E_2, \ldots, , E_{2n+1}$ such that $f(Ei) \cap g(Ei) = \emptyset$ for each $1 \leq i \leq 2n + 1$. This result has nice applications in digraphs, and a short proof was later given in [1].

Question: *Is it possible to generalise the above to any number of functions?*

More specifically, consider functions $f_1, \ldots, f_k$ from a set E into a set F such that $f_i(x) \neq f_j(x)$ for every $x \in E$ and $1 \leq i < j \leq k$. Suppose that there exists a positive integer n such that for any element z in F, there exists an integer $t \in \{1, \ldots, k\}$ such that $|f_t^{-1}(z)| \leq n$. Is it true that E can be partitioned into $2n + 1$ subsets of $E_1, \ldots, E_{2n+1}$ such that $f_p(E_i) \cap f_q(E_i) = \emptyset$ for each $1 \leq i \leq 2n + 1$ and $1 \leq p < q \leq k$?

References:

[1] Bessy, Stéphane, Frédéric Havet, and Etienne Birmelé. "Arc-chromatic number of digraphs in which every vertex has bounded outdegree or bounded indegree." Journal of Graph Theory 53.4 (2006): 315-332.

[2] El Sahili, Amine. "Functions and line digraphs." Journal of Graph Theory 44.4 (2003): 296-303.

Problem 2. *Reconfiguring* 6*-colorings of planar graphs with girth at least* 5 *(suggested by Carl Feghali)*

Background

The reconfiguration graph $R_k(G)$ for the k-colorings of a graph G has as vertex set all possible k-colorings of G and two vertices are adjacent if they differ in the color of exactly one vertex. In [1] and [2], we showed that for a planar graph G with n vertices, $R_{10}(G)$ has diameter $O(n)$. The first proof uses discharging and reducible configurations. The second proof uses a Thomassen-like approach. We further conjectured the following.

Conjecture: *If G is a planar graph with n vertices and girth at least 5, then $R_6(G)$ has diameter $O(n)$*

We believe that the conjecture is amenable for attack using the first proof method (i.e., discharging and reducible configurations) but probably not the second.

References:

[1] Dvořák, Zdeněk, and Carl Feghali. "An update on reconfiguring 10-colorings of planar graphs." arXiv preprint arXiv:2002.05383 (2020).

[2] Dvořák, Zdeněk, and Carl Feghali. "A Thomassen-type method for planar graph recoloring." arXiv preprint arXiv:2006.09269 (2020).

Problem 3. *Removal lemma for Latin squares (suggested by Jan Hladký)*

This is asking for a removal lemma for Latin squares. The removal lemma of Ruzsa and Szemeredi says that if the density of a fixed pattern (such as the triangle) in a graph is small then the graph can be made free of that pattern using only small changes. We ask whether the same phenomenon holds for Latin squares. Here, "patterns" are generated according to the following procedure.

Fix x and y. Sample x rows and y columns, and view them from left to right and from top to bottom, respectively. Look at the $x \cdot y$ many values seen at the intersection and replace them with their relative rank. So, this way, provided that all the initial values seen were different (which will be the case most of the time), we get one of $(xy)!$ many "patterns". To quantify properly, the removal lemma would state that for every $\varepsilon > 0$ there exists $\delta > 0$ so that if the number of a x-by-y patterns P in a Latin square of order n is less than $\delta n^{(x+y)}$ then by using at most εn^2 many changes we can turn the Latin square into a P-free Latin square.

Problem 4. *An extremal problem on crossing vectors (suggested by Tomas Juškevičius)*

Source: Proposed by Lason, Micek, Streib, Trotter and Walczak in 2014.

Definitions.
• *We say that two vectors $u, v \in \mathbb{Z}^d$ $(d \geq 2)$ are $k-$**crossing** if there are two coordinates i, j such that $u_i - v_i \geq k$ and $v_j - u_j \geq k$.*

Question: What is the maximum size of a family of vectors in $\mathbb{Z}^d$ that is 1-crossing, but not k-crossing?
Conjecture: the maximum size of such a family is k^{d-1}.

Related results:
• True for $d \leq 3$.
• For all d the following quantity is an upper bound:

$$k^d - k^2(k-1)^{d-2}$$

References:

M. Lason, P. Micek, N. Streib, W.Trotter and B. Walczak "An extremal problem on crossing vectors", *Journal of Combinatorial Theory, Series A* (2014).

Problem 5. *Edge modification criticality for H-free graphs (suggested by Adam Kabela)*

We recall that a graph is H*-free* if it contains no copy of H as an induced subgraph. We consider the class of all edge minimal H-free graphs (that is, graphs which are H-free but removing an arbitrary edge always creates an induced copy of H) and similarly consider the class of all edge maximal H-free graphs. What can we say about the intersection of the two classes for different choices of H?

We briefly thought about the question with Tomáš Kaiser, Théo Pierron and Kristýna Pekárková and observed that for various choices of H the intersection of the classes contains infinitely many graphs. (Clearly, understanding the intersection for H yields a similar understanding for the complement of H.) Furthermore, it is easy to see that the intersection is empty for H chosen as the path on four vertices (and trivially for H chosen as a complete graph or as a graph with no edges). Are there other graphs H for which the intersection is empty? In particular, it is non-empty for most graphs H on up to 5 vertices (for details, see picture below).

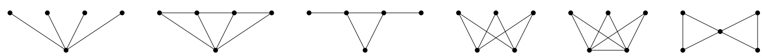

Figure 1: For these graphs on 5 vertices, we do not know the answer yet.

Problem 6. *Large touching matching (suggested by Tereza Klimošová)*

Source: Proposed by Paul Seymour in November 2019 at MATRIX Program: Structural Graph Theory Downunder.

Definitions.
• *A* touching matching *in a graph G is a matching M in G such that for all distinct $e, f \in M$, there is an edge of G joining an end of e and an end of f.*
• *The Hadwiger number $h(G)$ of a graph G is the size t of the largest complete graph K_t that is a minor of G.*
• *$\chi(G)$ denotes the chromatic number of G*

Conjecture 1 (Hadwiger's Conjecture)**.** *$\chi(G) \leq h(G)$.*

Let G be a graph with n vertices and no three-vertex stable set. Hadwiger's conjecture implies that there is a touching matching in G of size at least $\frac{n}{8} - 1$.

Question: Prove that a graph with n vertices and no three-vertex stable set has a touching matching of size at least $\frac{n}{1000}$.

Related results:
• Duchet, Meyniel [1]: For every n-vertex graph G, $h(G) \geq n/(2\alpha(G) - 1$. In particular, $h(G) \geq n/3$ when $\alpha(G) = 2$.
• See [2] for other results related to Hadwiger's conjecture.

References:
[1] P. Duchet and H. Meyniel, "On Hadwiger's number and the stability number", in Graph Theory (Proc. conf. on graph theory, Cambridge, 1981; B. Bollobás, ed.), Annals of Discrete Math. 13, North-Holland Mathematical Studies 62 (1982), 71–73.
[2] P. D. Seymour, "Hadwiger's conjecture", in Open problems in mathematics (Nash, J.F. and Rassias, M.T. eds.), 2016 New York: Springer, 417–448.

Problem 7. *Overpopulation rules in Simplified Game of Life (suggested by Michal Opler)*

Source: Proposed by Krishnendu Chatterjee, Rasmus Ibsen-Jensen, Ismaël Jecker and Jakub Svoboda in 2020.

Definitions.

• *A* configuration *of a graph G is a mapping of the vertices of G into the set of states $\{0, 1\}$. We say that a vertex is* dead *if it is in state* 0*, and* alive *if it is in state* 1*.*

• *A successor of a given configuration is determined by local* update rules*. We are interested in two simplified rule types.*

– Underpopulation rule $\mathcal{R}^+(i_0, i_1)$ – *a dead vertex becomes alive if it has at least i_0 live neighbors, and remains dead otherwise; a live vertex remains alive if it has at least i_1 live neighbors, and becomes dead otherwise.*

– Overpopulation rule $\mathcal{R}^-(i_0, i_1)$ – *a dead vertex becomes alive if it has at least i_0 live neighbors, and remains dead otherwise; a live vertex remains alive if it has at most i_1 live neighbors, and becomes dead otherwise.*

• *Given an update rule $\mathcal{R}$ and a graph G, the* configuration graph $\mathrm{C}(G, \mathcal{R})$ *is the (directed) graph whose vertices are the configurations of G, and whose edges are the pairs (c, c') such that the configuration c' is successor of c according to the update rule $\mathcal{R}$. Note that $\mathrm{C}(G, \mathcal{R})$ is finite since G is finite. Moreover, since the update rule $\mathcal{R}$ is deterministic, every vertex of the configuration graph is the source of a single infinite walk composed of a finite path followed by a cycle.*

– *The* configuration reachability problem*, denoted* Reach*, asks, given a graph G, an initial configuration c_I, and a final configuration c_F, whether the walk in $\mathrm{C}(G, \mathcal{R})$ starting from c_I eventually visits c_F.*

– *The* long-run average problem*, denoted* Avg*, asks, given a threshold $\delta \in [0, 1]$, a graph G, and an initial configuration c_I,* *whether δ is strictly smaller than the average ratio of live vertices in the configurations that are part of the cycle in* $\mathrm{C}(G, \mathcal{R})$ *reached from* c_I.

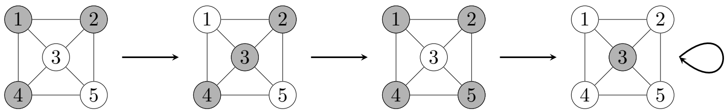

Figure 2: Evolution of a graph under the overpopulation rule $\mathcal{R}^-(3,1)$. Live vertices are gray.

Question: How hard are Reach and Avg when the underlying graphs belong to a restricted graph family - for instance two-dimensional grids, or planar graphs?

Related results:
• Chatterjee et al. [1] showed that for any underpopulation rule both Reach and Avg can be decided in polynomial time.
• On the other hand, they proved that for the overpopulation rule $\mathcal{R}^-(2,1)$ both Reach and Avg are PSPACE-complete. This remains true even when the underlying graphs are restricted to regular graphs of degree 10.

References:
[1] Krishnendu Chatterjee, Rasmus Ibsen-Jensen, Ismaël Jecker and Jakub Svoboda. Simplified Game of Life: Algorithms and Complexity. *45th International Symposium on Mathematical Foundations of Computer Science (MFCS 2020)*, 2020.

Problem 8. *Finding formulas for betweenness centrality in specific graph classes (suggested by Aneta Pokorná)*

Definitions.

- *A path on* k *vertices* $p_1, \ldots, p_k$ *is denoted by* P_k*. Let* C_k *be a cycle on* k *vertices* $c_1, \ldots, c_k$*.*
- *Cartesian product* $G \square H$ *of two graphs* G, H *is a graph with vertex set* $V(G) \times V(H)$*. Vertices* $(x, u), (y, v)$ *are adjacent in* $G \square H$ *whenever* $x = y$ *and* $uv \in E(H)$ *or* $u = v$ *and* $xy \in E(G)$*.*
- *Wheel* W_k *is a graph created from a cycle* C_k *by adding a central vertex* c *adjacent to all vertices of the cycle.*
- *Subdivided wheel* SW_k *is created from a wheel graph by subdividing edges incident to the central vertex.*

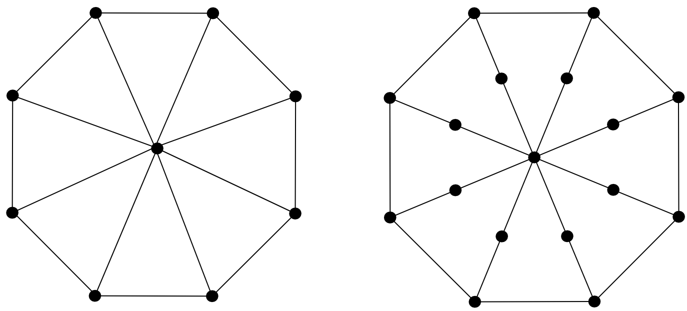

Figure 3: Wheel W_8 and subdivided wheel SW_8

- *Spider net* $SN_{k,\ell}$ *is a graph created from* $P_k \square C_\ell$ *by adding a new vertex* c *and connecting it to vertices* (p_1, c_i) *for all* $i \in \{1, \ldots, \ell\}$*.*
- *Sunflower graph* SF_k *is a graph created from wheel* W_k *by adding new vertices* $n_1, \ldots, n_k$ *where* n_i *is adjacent to* c_i *and* $c_{i+1 \mod k}$*, where* c_i *for all* $i \in \{1, \ldots, k\}$ *are the vertices of the outer cycle of the wheel.*
- *Gear graph* G_k *is created by subdividing each edge of the outer cycle of the wheel graph* W_k*.*

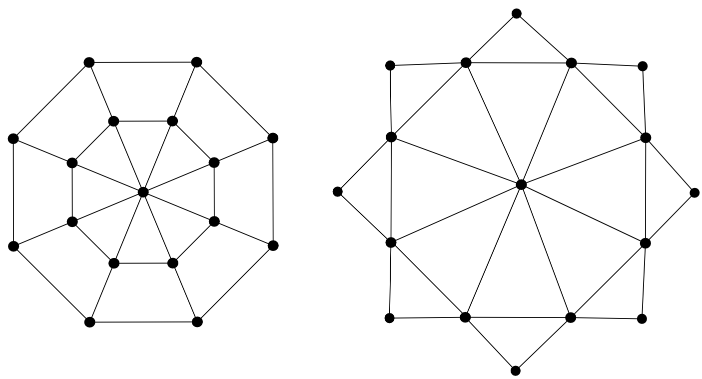

Figure 4: Spider net $SP_{2,8}$ and sunflower SF_8

• *Betweenness centrality* $B(u)$ *of a vertex* u *measures the importance of vertex* u *based on the number of shortest paths passing through* u*. It is defined as*

$$B(u) = \sum_{x,y \in V(G), x \neq u \neq y} \frac{\sigma_{x,y}(u)}{\sigma_{x,y}} \tag{1}$$

where $\sigma_{x,y}$ *is the total number of shortest* x, y*-paths and* $\sigma_{x,y}(u)$ *is the total number of shortest* x, y*-paths going through* u *(not including paths with an endpoint in* u*).*

Question: Determine betweenness centrality of all types of vertices in

- subdivided wheels
- suflowers with subdivided edges incident to the central vertex
- sunflowers with subdivided edges of the main cycle

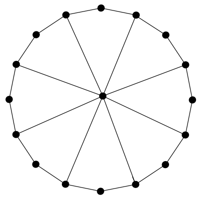

Figure 5: Gear graph G_8

- spider nets
- grids $P_m \Box P_n$

using reasonable parameters such as

- degree of the central vertex
- order of the graph
- dimensions of the grid

Related results:

Theorem 2 (Raghvan Unnithan, Kannan, Jathavedan, 2014, Aytac, 2017)**.** *For any $v \in V(W_k)$ of order $n = k+1$ with $k \geq 6$ it holds*

$$B(v) = \begin{cases} \frac{k(k-4)}{2} = \frac{(n-1)(n-5)}{2}, & \text{if } v \text{ is the central vertex} \\ \frac{1}{2}, & \text{otherwise} \end{cases}$$

Theorem 3 (Aytac, 2017)**.** *For $k \geq 5$, let G_k of order $n = 2k+1$*

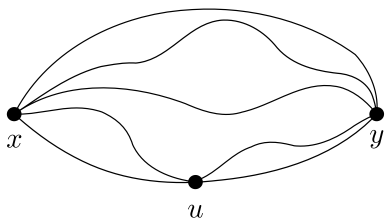

Figure 6: Illustration of the contribution of pair of vertices x, y to betweenness centrality of vertex u. Curves symbolize shortest paths. The number of shortest paths from x to y thorough u is four, as there are two ways how to get from x to u and two ways how to get from u to y. The contribution of x and y to betweenness of u is $\frac{4}{4+3} = \frac{4}{7}$, as there are four shortest paths between x and y through u and seven shortest paths between x and y in total.

with central vertex denoted by c. It holds

$$
\begin{aligned}
B(c) &= 2k^2 - \frac{161k}{30} = \frac{(n-1)^2}{2} + \frac{161(n-1)}{60} \\
B(v) &= 2k - 3 \qquad = n - 4 && \text{if } \deg(v) = 3 \\
B(v) &= \frac{41}{30} && \text{if } \deg(v) = 2
\end{aligned}
$$

Theorem 4 (Aytac, 2017)**.** *Let SF_k be a sunflower on $n = 2k+1$ vertices with central vertex c. Then*

$$
\begin{aligned}
B(c) &= 2k^2 - \frac{281k}{30} = \frac{(n-1)^2}{2} - \frac{281(n-1)}{60} \\
B(v) &= 2k - \frac{49}{30} \qquad = n - \frac{79}{30} && \text{if } \deg(v) = 5 \\
B(v) &= 0 && \text{if } \deg(v) = 2
\end{aligned}
$$

Proposition 5 (Sunil Kumar, Balakrishnan, 2019)**.** *Take a vertex x in $P_m \square P_n$; then $x = (a, b)$, where $1 \leq a \leq m$, $1 \leq b \leq n$. The*

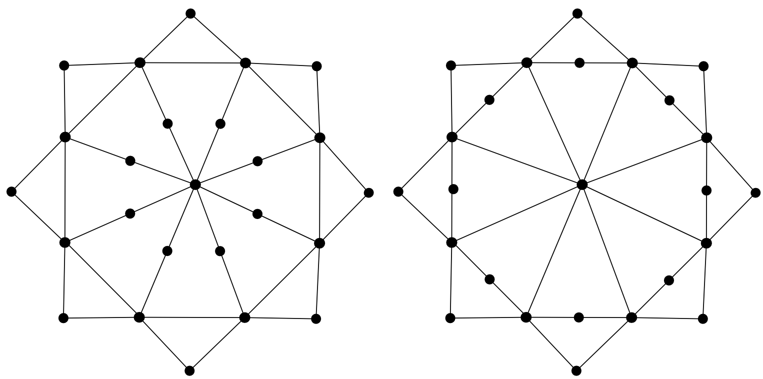

Figure 7: Sunflower SF_8 with subdivided edges incident to the central vertex and with subdivided edges of the main cycle

paths P_n^a and P_m^b passing through (a, b) divide the rectangular grid into four quadrants A, B, C, D sharing their common sides. Any pair of vertices lying in the diagonal regions A, B or C, D makes a contribution to the betweenness centrality of (a, b). Hence

$$B(x) = B[(a, b)] =$$

$$= \sum_{u,v} \frac{\sigma(u, x)\sigma(x, v)}{\sigma(u, v)} + \sum_{w,z} \frac{\sigma(w, x)\sigma(x, z)}{\sigma(w, z)} - [(a-1)\cdot(m-a)+(b-1)\cdot(n-b)]$$

where $u \in A, v \in B, w \in C, z \in D$.

Proposition 6 (Sunil Kumar, Balakrishnan, 2019)**.** *In a grid $P_n \Box P_m$, for any inner vertex $w_0 = (a, b)$, any vertex at a distance $d = \min(a, b, m - a, m - b) > 0$ from w_0 induces a contribution $4d$ to $B(w_0)$.*

Proposition 7 (Sunil Kumar, Balakrishnan, 2019)**.** *In a grid $P_m \Box P_n$, for any vertex $w_0 = (a, b)$, $1 < a < m$, $1 < b < n$, the k-neighbourhood of w_0 where $k = \min(a, b, m - a, m - b) > 0$ denoted*

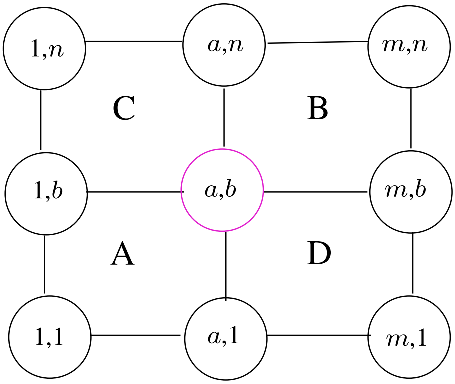

Figure 8: Illustration to proposition 5.

by $N_k(w_0)$ contains $2k(k+1)$ vertices and the contribution to the betweenness centrality of w_0 induced by $N_k(w_0)$ is $2k^2(k+1)$.

References:

R. S. Kumar and K. Balakrishnan. Betweenness centrality in Cartesian prod- uct of graphs. *AKCE International Journal of Graphs and Combinatorics*, Mar 2019. doi: 10.1016/j.akcej.2019.03.012

V. Aytac. On the centrality of some graphs. *New Trends in Mathematical Science*, 4:1–11, Okt 2017. doi: 10.20852/ntmsci.2017.209.

R. S. Kumar, K. Balakrishnan, and M. Jathavedan. Betweenness centrality in some classes of graphs. *International Journal of Combinatorics*, 2014. ISSN 1687-9163. doi: 10.1155/2014/241723.

Problem 9. *Star coloring (suggested by Robert Šámal)*

Source: Proposed by Z. Dvořák, B. Mohar and R. Šámal

The star chromatic index $\chi'_s(G)$ of a graph G is the minimum number of colors needed to properly color the edges of the graph so that no path or cycle of length four is bi-colored.

Question: What is $\chi'_s(K_n)$?

Conjecture: If G is a subcubic graph then $\chi'_s(G) \leq 6$.

Related results:

• [DBŠ] obtain a near-linear upper bound in terms of the maximum degree $\Delta = \Delta(G)$. In particular,

$$\chi'_s(K_n) \leq n \cdot \frac{2^{2\sqrt{2}(1+o(1))\sqrt{\log n}}}{(\log n)^{1/4}}.$$

or more crudely, for every $\varepsilon > 0$ there exists a constant c such that $\chi'_s(K_n) \leq cn^{1+\varepsilon}$ for every $n \geq 1$.

• [DBŠ] $\chi'_s(K_n) \geq 2n(1 + o(1))$.

• [DBŠ] If G is a subcubic graph, then $\chi'_s(G) \leq 7$. If G is a simple cubic graph, then $\chi'_s(G) \geq 4$, and the equality holds if and only if G covers the graph of the 3-cube.

• [BLMSS] If G is a cubic outerplanar graph or a cubic tree then $\chi'_s(G) \leq 6$.

• [GGR] If G is a cubic Halin graph then $\chi'_s(G) \leq 6$.

• [GGR] If G is a bipartite graph where one part has max. degree 2 and the other part max. degree 3, then $\chi'_s(G) \leq 5$

References:

• L. Bezegova, B. Luzar, M. Mockovciakova, R. Sotak, R. Skrekovski, Star edge coloring of some families of graphs, Journal of Graph Theory 81 (2016), 73–82.

• C. J. Casselgren, J. B. Granholm, A. Raspaud: On star edge colorings of bipartite and subcubic graphs, https://arxiv.org/abs/1912.02467

• and references therein

• Z. Dvořák, B. Mohar, R. Šámal: Star chromatic index, J. Graph Theory 72 (2013), 313–326

Problem 10. *How dense must a random graph be so that at each vertex we see approximately the same number of trees of a given shape? (suggested by Matas Šileikis)*

The *binomial random graph* $\mathbb{G}(n, p)$ is a graph on n labeled vertices obtained by connecting each each pair $i < j$ of vertices independently with probability p. We allow $p = p(n)$ and study properties of $\mathbb{G}(n, p)$ that hold w.h.p. (with high probability), that is with probability tending to 1, as n grows.

Fix a *rooted tree* T with k edges (say, a path of length k with one endpoint being the root vertex).

Let $X_{n,v}$ be the number of trees in $\mathbb{G}(n, p)$ which are rooted at v and are isomorphic to T. Let's call such trees T-*extensions* of v. In the simplest case where T is a single edge, we have $X_{n,v} = \deg_{\mathbb{G}(n,p)}(v)$. So we can interpret the vector $(X_{n,1}, \ldots, X_{n,n})$ as a generalization of the degree sequence.

We want to understand when the number T-extension are approximately the same for every vertex. This problem, for more general rooted graphs (not just trees) was considered by Spencer (1990) and refined by Šileikis and Warnke (2019+).

We will assume $np \geq \log n$, since otherwise (an exercise!) with positive probability there is an isolated vertex, so we cannot even expect that $X_{n,v} \geq 1$ w.h.p.

Further, we fix a sequence $\varepsilon = \varepsilon(n) > 0$ that slowly tends to 0. The question is: *for what sequences $p = p(n, \varepsilon)$ is it true that*

$$\mathbb{P}\left\{\forall \text{ vertex } v \quad X_{n,v} = \mathbb{E}X_{n,v}(1 \pm \varepsilon)\right\} \to 1, \qquad \text{as } n \to \infty? \tag{2}$$

Example: If it looks scary, let's look at the simplest case when T is an edge, so $X_{n,v}$ is a binomial random variable (thus with expectation $(n-1)p \sim np$). One of the many inequalities which is called Chernoff implies

$$\mathbb{P}\left\{|X_{n,v} - (n-1)p| \geq \varepsilon np\right\} \leq \exp\left\{-\frac{\varepsilon^2 np}{2 + o(1)}\right\}. \tag{3}$$

(Here we assume $\varepsilon^2 np \to \infty$, that is what "$\varepsilon$ slowly tends to 0" means in this case.) If, we require p to satisfy

$$\varepsilon^2 np \geq (2+\delta)\log n, \tag{4}$$

for some fixed $\delta > 0$, then inequality (3), combined with the union bound, implies that

$$\max |X_{n,v} - (n-1)p| \leq \varepsilon np \text{ w.h.p.}$$

The constant 2 in (4) is optimal. It's not too hard to show that if

$$\varepsilon^2 np \leq (2-\delta)\log n,$$

then

$$\max |X_{n,v} - (n-1)p| > \varepsilon np \text{ w.h.p.}$$

Hoping that for a general tree T the situation is similar, we reformulate our question as follows.
Question: For which rooted trees T, there is a constant $C = C(T) > 0$ such that for any constant $\delta > 0$

$$\mathbb{P}\left\{\forall \text{ vertex } v \quad X_{n,v} = \mathbb{E}X_{n,v}(1 \pm \varepsilon)\right\} \to \begin{cases} 1, & \text{if } \varepsilon^2 np \geq (C+\delta)\log n \\ 0, & \text{if } \varepsilon^2 np \leq (C-\delta)\log n. \end{cases} \tag{5}$$

References:

Šileikis, M., & Warnke, L. (2019). Counting extensions revisited. *arXiv preprint* arXiv:1911.03012.

Spencer, J. (1990). Counting extensions. *Journal of Combinatorial Theory, Series A*, 55(2), 247-255.

Problem 11. *Monochromatic antipodal paths in hypercubes (suggested by Tung Anh Vu)*

Source: S. Norine [1]

Definitions.
• *Let* $\Delta_H(u, v)$ *denote the* Hamming distance *between vectors* $u, v \in \mathbb{Z}_2^n$*. An* n-dimensional hypercube Q_n *is an undirected graph with vertex set* $V(Q_n) = \mathbb{Z}_2^n$ *and edge set*

$$E(Q_n) = \{\{u, v\} : \Delta_H(u, v) = 1\}.$$

• *By a* 2*-coloring of a hypercube we mean any function* $c : E(Q_n) \to \{0, 1\}$*.*
• *For a vertex* $u = (u_1, \ldots, u_n) \in V(Q_n)$ *its* antipodal vertex u' *is the vertex* $(1 - u_1, \ldots, 1 - u_n)$*. For an edge* $e = \{u, v\} \in E(Q_n)$ *its* antipodal edge *is the edge* $e' = \{u', v'\}$*.*
• *An* antipodal coloring *of a hypercube is a coloring such that each pair of antipodal vertices have opposing colors.*

Example: A properly colored $Q_2 \simeq C_4$ is not antipodally colored and does not have such a pair. However an antipodally colored Q_2 has such a pair, hence the motivation for an antipodal coloring.
Question: Let G be a hypercube Q_n. Is there a pair of antipodal vertices connected by a monochromatic path for each antipodal coloring c?

Related results:
• This conjecture has been verified for $n \leq 6$.
• The following conjecture by Feder and Subi [2] is equivalent to Norine's conjecture: Is there a pair of antipodal vertices connected by a path which changes colors at most once for any 2-coloring of Q_n?
• See `http://reu.dimacs.rutgers.edu/~tv157/topic-presentation.pdf` and `http://reu.dimacs.rutgers.edu/~tv157/final-presentation.pdf`.

References:

[1] Edge-antipodal colorings of cubes. The Open Problem Garden, Norine, S, 2008.

[2] On hypercube labellings and antipodal monochromatic paths, Feder, Tomás and Subi, Carlos, Discrete Applied Mathematics, 161, 10-11, 1421–1426, 2013, Elsevier.