Algorithms and datastructures II

Lecture 1: text searching (1/2)

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Prague

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Algorithms and data-structures II

Lecturer: Jan Hubička, `hubicka@kam.mff.cuni.cz`.

- Consultations by appointment (email or zoom).
- Do we want a discussion forum, like discord or moodle?

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Lecture: Monday 9am over zoom.

- Lectures will be live: please come and do ask questions, tell me how you like the lecture!
- Recordings will be available on `stream.cuni.cz` (linked from webpage).
- Recordings will include all questions and communication. If you ask me after the lecture ends, I will edit out your question.
- I will try to keep lecture synchronized with Czech lecture by Jan Hric.
- If you spot mistake, please let me know. The slides are brand new.

Webpage: `https://iuuk.mff.cuni.cz/~hubicka/2020/adsII.html`

Syllabus

1. String-searching

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2. Network flows

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3. Algebraic algorithms (Fourier transformation)

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4. Parallel algorithms

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5. Geometric algorithms
6. Introduction to complexity
7. Approximation algorithm
8. Probabilistic algorithms and cryptography

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8. $\alpha[i:j]$: subword $\alpha[i]\alpha[i+1]\cdots\alpha[j-1]$
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10. $\alpha[i:]$: a suffix of α
11. $\alpha[:]$: whole word α

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Occurrence of ν in η is any index i such that $\eta[i : i + |\nu|] = \nu$

Naive approach

string-searching

Given a string ν (“a needle”) and η (“a haystack”) find all occurrences of ν in η.

Search (ν, η):

1. For $i = 0, \ldots, |\eta| - |\nu| - 1$:
2. If $\eta[i : i + |\nu|] = \nu$: output i

$\nu = \textit{coconut}$

η is some very long text about coconuts

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Time complexity: $\Theta(|\nu| \cdot |\eta|)$.

Incremental algorithm

An incremental algorithm receives characters of η one by one and after receiving a new character it immediately outputs possible new occurrences of ν.

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Basic idea

We would like to remember an state. This is longest prefix of ν which is a suffix of η.

Observation: Whenever algorithm enters state ν it finds a new occurrence of ν in the input.

Assume that algorithm seen string η, is in state α and receives a new character c. How to update the state?

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$$\text{State}(\alpha, c) = \begin{cases} \alpha c & \text{If } \alpha c \text{ is a prefix of } \nu \\ & \end{cases}$$

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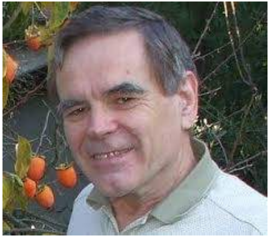
$$\text{State}(\alpha, c) = \begin{cases} \alpha c & \text{If } \alpha c \text{ is a prefix of } \nu \\ \epsilon & \\ \alpha' c & \alpha' c \text{ is a prefix of } \nu \text{ and } \alpha' \text{ is a suffix of } \alpha \end{cases}$$

We want to compute backward function b which tells for every prefix α of ν the longest proper suffix α' (of α) that is also a prefix of ν.

Knuth–Morris–Pratt (KMP) algorithm (1974)



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Searching automaton

1. State $0, \ldots, |\nu|$
   (state s corresponds to prefix $\nu[: s]$)
2. Forward edges: $s \to s + 1$
3. Backward edges: pointing from $s > 0$ to j such that $\nu[: j]$ is a proper suffix of $\nu[: s]$

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Step (s, c):

1. While $s \neq 0$ and $\nu[s] \neq c$:
2. $\quad s \leftarrow b[s]$.
3. If $\nu[s] = c$: $s \leftarrow s+1$.
4. Return s

Search (η, automaton for ν):

1. $s \leftarrow 0$.
2. For $i = 0, \ldots, |\eta| - 1$:
3. $\quad s \leftarrow$ Step $(s, \eta[i])$.
4. $\quad$ If $s = |\nu|$: report $i - |\nu| + 1$.

Invariant: The state s corresponds to the longest suffix of $\eta[:i]$ that is a prefix of ν.

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Lemma

Search will run in time $O(|\eta|)$

Proof.

1. Search itself is clearly $O(|\eta|)$

□

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Overall runtime $\Theta(|\eta|)$.

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How to obtain the automaton?

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To determine $b(s)$ we need to search $\nu[1:s]$
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Search $(\eta$, automaton for $\nu)$:

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KMPConstruction (ν):

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4. $\quad b[i] \leftarrow s$.

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Theorem

Algorithm KMP will finish in time $\Theta(|\eta| + |\nu|)$.

Invariant: The state s corresponds to the longest suffix of $\eta[: i]$ that is a prefix of ν.

Rabin–Karp algorithm (1987)

Recall the rotating/sliding hash function

$$H(x_1, x_2, \ldots, x_K) = (x_1 P^{K-1} + x_2 P^{K-2} + \cdots + x_{K-1} P^1 + x_K P^0) \mod N$$

For $K = |\nu|$, prime number P and $N > 0$.

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RabinKarpSearch (ν, η):

1. Choose prime P and $N > 1$.
2. Precompute $P^{|\nu|} \mod N$.
3. $n \leftarrow H(\nu)$.
4. $h \leftarrow H(\eta[: |\nu|])$.
5. For $i = 0, \ldots |\eta| - |\nu| - 1$:
6.   if $h = n$ and $\eta[i : i + |\nu|] = \nu$: report i.
7. $h \leftarrow (P \cdot h - \eta[i] \cdot P^{|\nu|} + \eta[i + K]) \mod N$.

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7. $h \leftarrow (P \cdot h - \eta[i] \cdot P^{|\nu|} + \eta[i + K]) \mod N$.

Expected time complexity: $O(|\nu| + |\eta| + C|\nu| + |\eta|/N \cdot |\nu|)$ where C is number of occurences reported.