Combinatorics and Graph Theory II (exercise sheet 2)

Name:

Nick:

Exercise 1, 2 points: Let F be a 3-regular, vertex 2-connected planar graph. Show that G has edge chromatic number 3. You can use, without a proof, the four colour theorem. (Thus you can assume that every planar graph has chromatic number at most 4.)

Exercise 2, 2 points: In the following I consider two isomorphic graphs as equal. More formally "graph" is not one specific graph but a whole class of mutually isomorphic graphs (more precisely I this speak about isomorphism types rather than graphs).

Let $G \preceq_M H$ be a relation G *is a minor of* H. Let $\mathcal{F}$ be a set of graphs. Denote $\mathrm{Forb}(\mathcal{F})$ set of all graphs not containing any graph from $\mathcal{F}$ as a minor (for example, $\mathrm{Forb}(K_5, K_{3,3})$ is the class of all planar graphs as given by Kuratogwski-Wagner theorem).

Let M be a set of graphs. Prove that the following statements are equivalent:

1. Set M is minor-closed. This means that every for every graph G belonging to M it holds that also all minors of G belongs to M.
2. There exists a set of graphs $\mathcal{F}$ (it may be infinite) such that $M = \mathrm{Forb}(\mathcal{F})$. ($\mathcal{F}$ is usually called the set of forbidden minnors for M.)

Exercise 3, 2 points: Show that every d-degenerated graph has average degree at most $2d$.

Exercise 4, 2 points: Show that for every surface Γ there are only finitely many non-isomorphic 7-regular graphs that can be embedded to Γ.

Exercise 5, 2 points: Show that every graph with chromatic number k has at least $\binom{k}{2}$ edges.

Exercise 6, 3 points: Find all surfaces Γ for which there exists infinitely many mutually non-isomorphic 6-regular graphs that can be embedded to Γ.

Please hand in your solutions before **Monday December 3** either in paper form or by email to **hubicka@kam.mff.cuni.cz**. Please add **CG-II** into the subject.