NMAG403 - Combinatorics

December 6, 2024 – Diverse Topics, Last Set of Problems

Homework

Deadline: **January 3, 2025**
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1. Let G be a cubic graph with a Hamiltonian cycle. Prove that G has at least 3 Hamiltonian cycles.
2. Show that every complete graph with an even number of vertices is Vizing class 1. (I.e., $\chi'(K_{2n}) = 2n - 1$ for every positive integer n.)
3. For a graph G, let $\zeta(G)$ be the smallest integer k such that G has an $\mathcal{L}$-list coloring for every list assignment $\mathcal{L} = \{L(u), u \in V(G)\}$ satisfying

   (a) $|L(u)| = k$ for every $u \in V(G)$, and

   (b) $|L(u) \cap L(v)| \leq 1$ for every edge $uv \in E(G)$.

   Prove that there are infinitely many values of n such that $\zeta(K_n) > \sqrt{n}$.

In class problems

45. Let G be a graph on n vertices and let $\bar{G}$ be its complement (i.e. uv is an edge of $\bar{G}$ if and only if it is not an edge of G). Prove that $ch(G) + ch(\bar{G}) \leq n + 1$, where ch denotes the chooseability.
46. Show that the edges of a complete graph with an odd number of vertices can be partitioned into disjoint Hamiltonian cycles.
47. A spanning subgraph H (not necessarily induced) of a graph G is called a *k-factor* of G if $\deg_H u = k$ for every $u \in V(H) = V(G)$. Prove the following: If k is a positive integer and a $2k$-regular graph G has an even number of edges, then G contains a k-factor. Show that the assumption on the parity of the number of edges of G is necessary.
48. A spanning subgraph H (not necessarily induced) of a graph G is called a *k-factor* of G if $\deg_H u = k$ for every $u \in V(H) = V(G)$. Prove the following: If k is a positive integer, then the edges of any $2k$-regular graph G can be partitioned into k disjoint 2-factors of G.