

# NMAG403 - Combinatorics

October 25, 2024 – Matchings in graphs

## In class problems

22. Prove the **Tutte theorem with defect**: For a positive integer  $d$ , a graph  $G$  contains a matching that misses at most  $d$  vertices if and only if for every set  $A$  of vertices of  $G$ , it holds true that

$$c_{\text{odd}}(G - A) \leq |A| + d.$$

(Hint: First prove the theorem for the case when  $d$  and  $|V(G)|$  are of the same parity.)

23. Design a polynomial time algorithm for constructing an Edmonds forest in an input graph.
24. The  $b$ -FACTOR problem is the problem to decide if an input graph  $G$  has a spanning subgraph  $H$  such that  $\deg_H u = b(u)$  for every vertex  $u \in V(G)$ , where  $b : V(G) \rightarrow \{0, 1, 2, \dots\}$  is a function also given as part of the input. Show that  $b$ -FACTOR is polynomial time solvable.
25. Let BOUNDED-DEGREE-SUBGRAPH denote the problem which asks whether an input graph  $G$  has a spanning subgraph  $H$  such that  $a(u) \leq \deg_H u \leq b(u)$  for every vertex  $u \in V(G)$ , where  $a, b : V(G) \rightarrow \{0, 1, 2, \dots\}$  are functions also given as part of the input. Decide if BOUNDED-DEGREE-SUBGRAPH is also polynomial time solvable, or NP-complete.