NDMI028 - LAK

October 10, 2024 – Linear dependence and independence of vectors

In class problems

12. Recall that an Even-Odd-ton is a set family such that each set has odd size and intersections of different sets are of even size. An Even-Odd-ton is *maximal* if no set can be added to it without violating the rules and without adding new citizens (elements). Do all maximal Even-Odd-tons with n citizens have the same size?
13. An Odd-Even-ton is a set family such that each set has even size and intersections of different sets are of odd size. Determine the maximum size of an Odd-Even-ton with n citizens.
14. Do all maximal Odd-Even-tons with n citizens have the same size?
15. For an integer s, a Mod-s-ton is a set family such that no set has size divisible by s while the sizes of the intersections of different sets are all divisible by s. Show that for every s, there exists a constant $c(s)$ such that every Mod-s-ton with n citizens has size at most $c(s) \cdot n$.
16. Denote by $m_s(n)$ the maximum size of an s-distance point set in R^n. Prove that

$$\binom{n+1}{2} \leq m_2(n).$$

17. Prove that for $s \geq 3$,

$$\binom{n+1}{s} \leq m_s(n) \leq \binom{n+s+1}{s}.$$

18. A set P of points in R^n is called *spherical* if the points belong to the surface of a sphere in R^n. Denote by $m_s^{sph}(n)$ the maximum size of an s-distance spherical point set in R^n. Prove that

$$\binom{n+1}{2} \leq m_2^{sph}(n) \leq \frac{n(n+3)}{2}.$$