RepExt and SimRep on Geometric Contact Graphs

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MCW 2025

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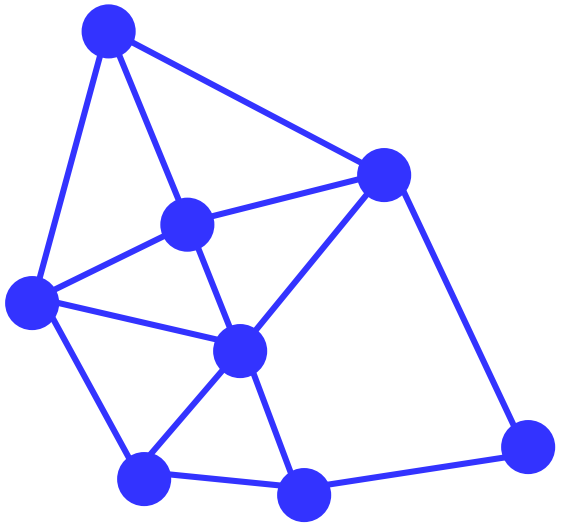
Kissing lemma, coin representations (Koebe 1936)



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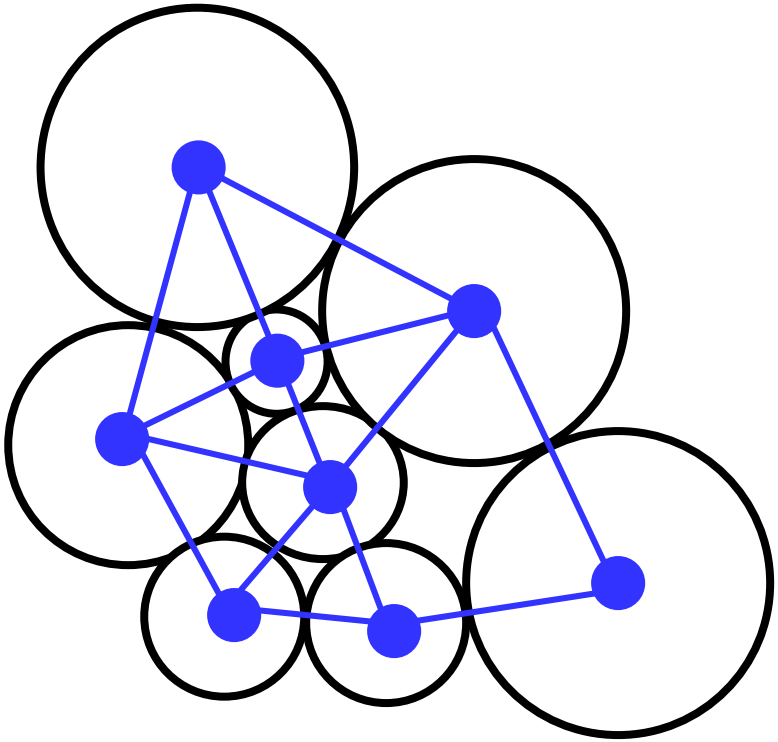
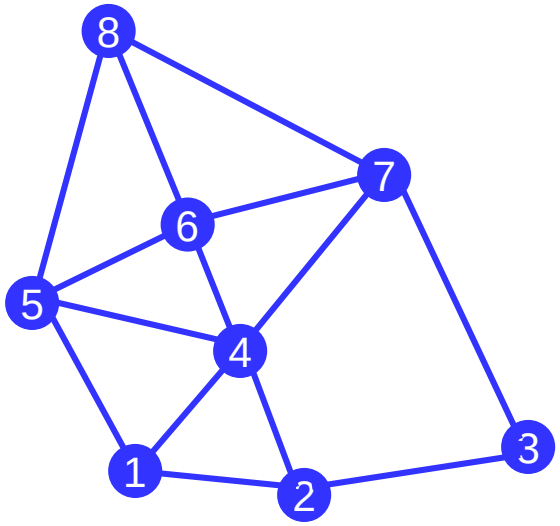
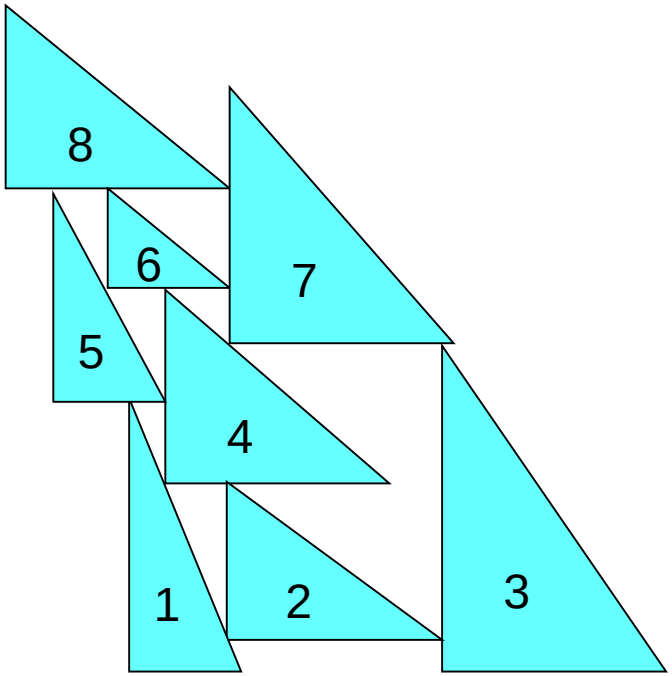


de Fraysseix, Ossona de Mendez, Rosesthiel (1997): Every planar graph is a contact graph of triangles.





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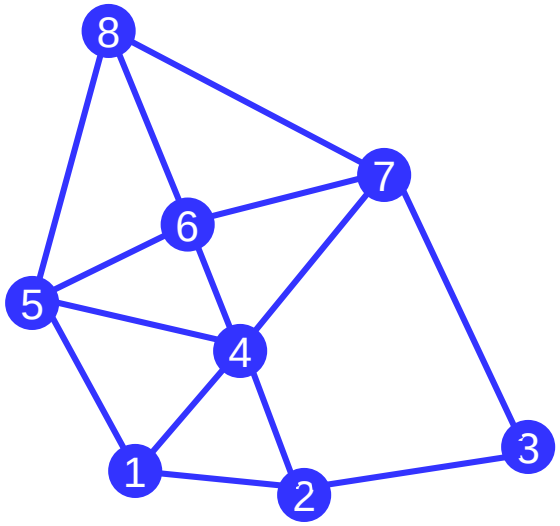
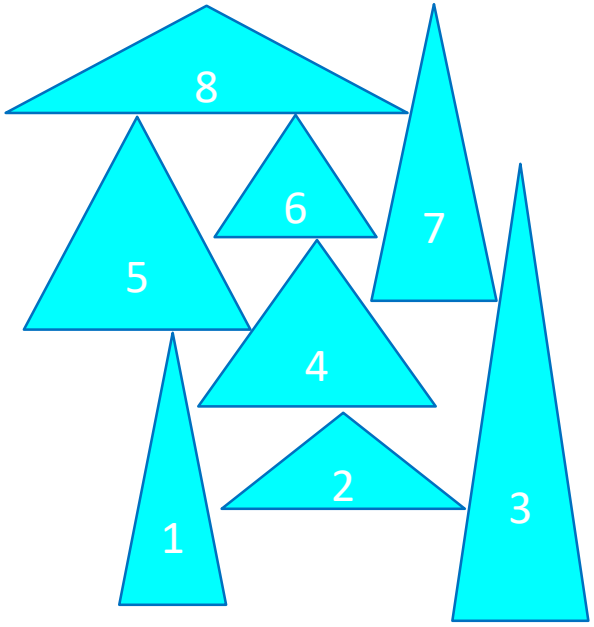
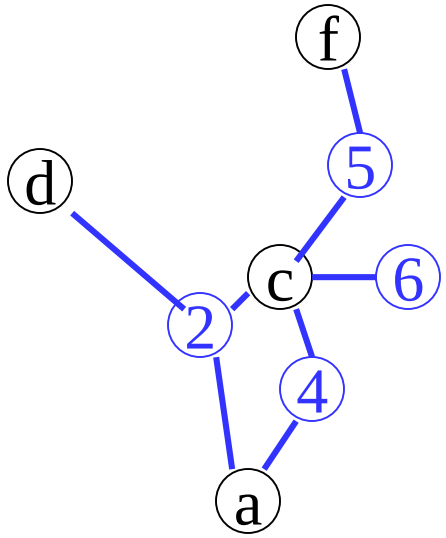
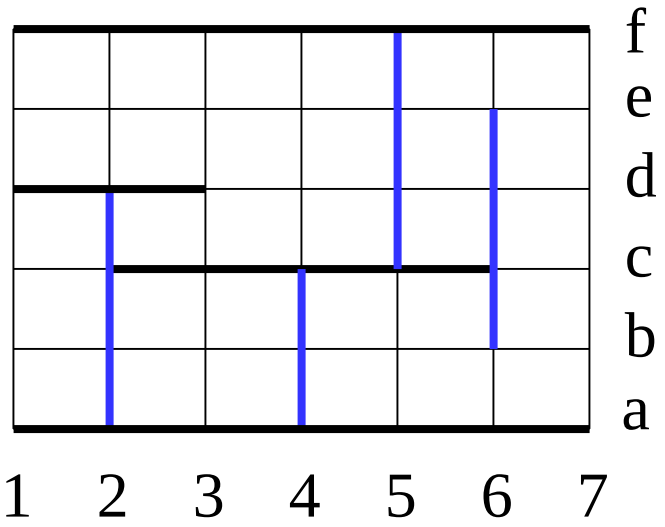




de Fraysseix, Ossona de Mendez, Pach (1994): Bellantoni, Ben-Arroyo Hartman, Przytycka, Whitesides (1993): Every planar bipartite graph is a contact graph of vertical and horizontal segments.





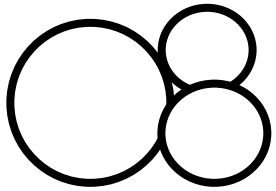
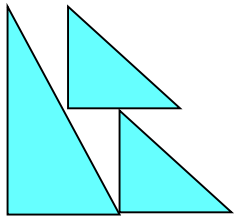
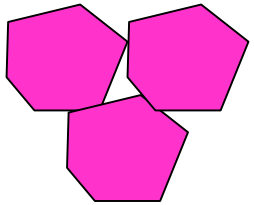
Extending

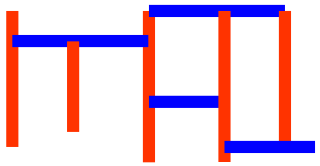
Partial Contact Representations

of

Planar Graphs

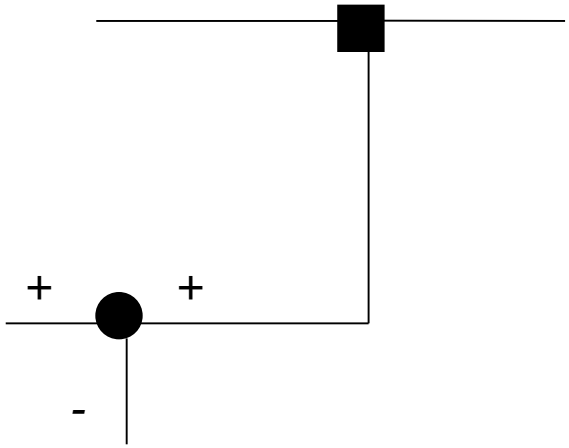
Thm 1 (Chaplick, Dorbec, JK, Montassier, Stacho – WG 2014): Given a planar graph and a partial contact representation by discs (triangles, convex sets, segments, grid segments), it is NP-hard to decide if the representation can be extended.



NP-hardness proof
Grid segments

Reduction from Planar Satisfiability (clauses of size 2 or 3, variables of degree 3)



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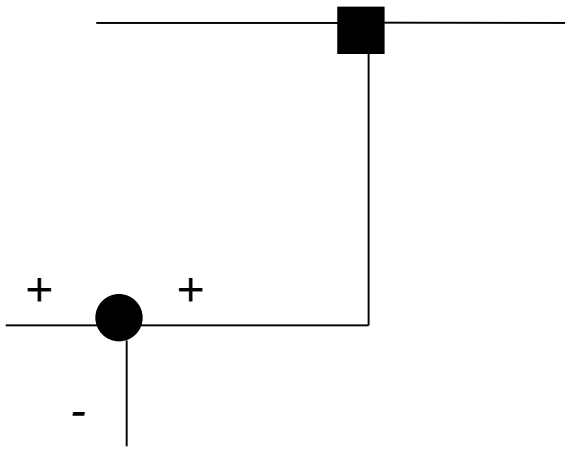
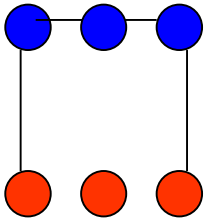
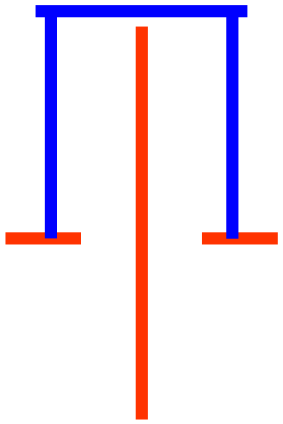


1. Given a formula Φ
2. Consider noncrossing rectilinear drawing
3. Replace variables/clauses/ occurrences by var/cl/edge gadgets to obtain G s.t.
4. G extendable iff Φ satisfiable

Key building block





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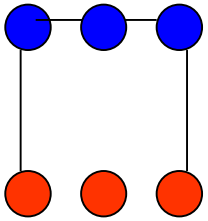

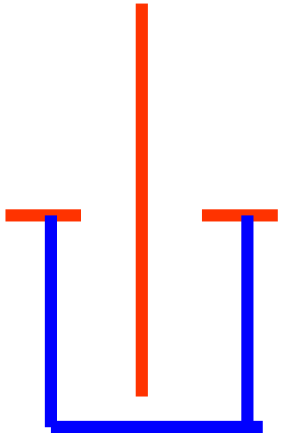

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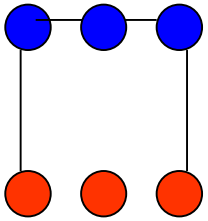

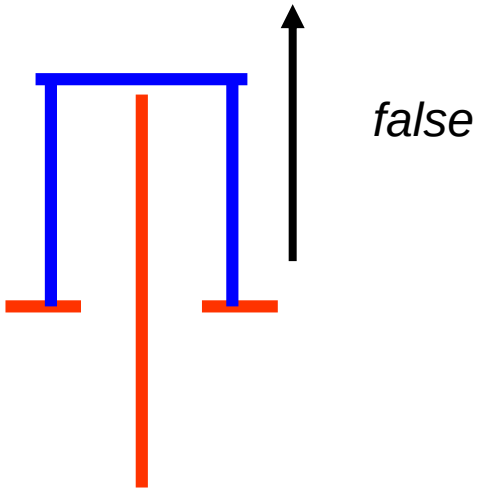

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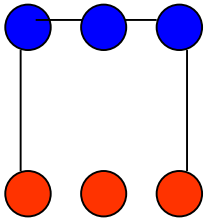
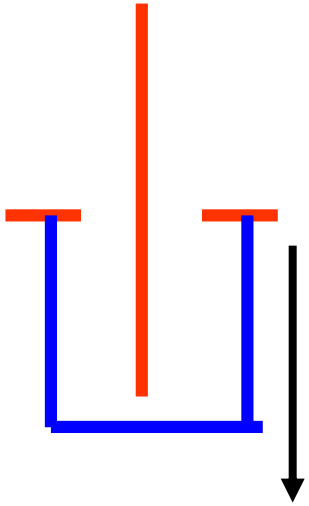
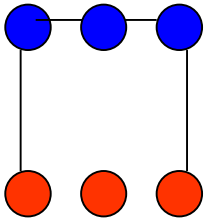
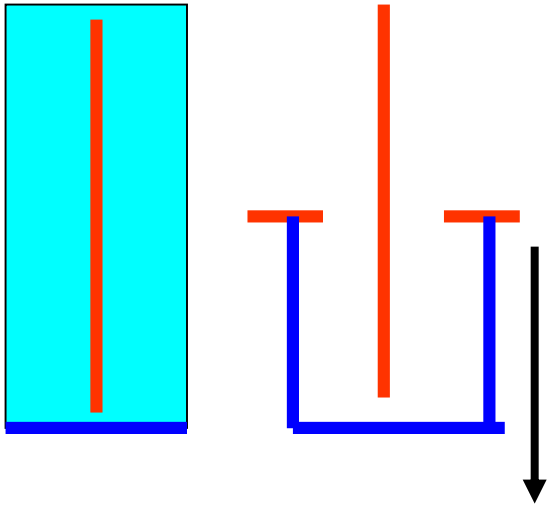




false

Key building block





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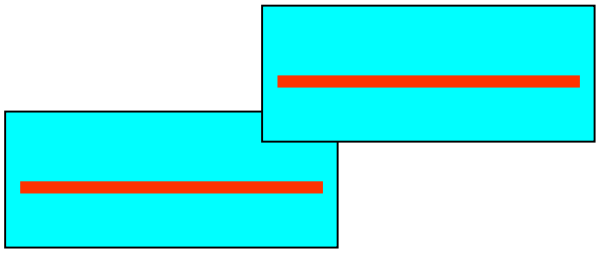
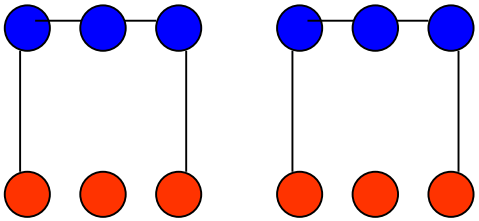
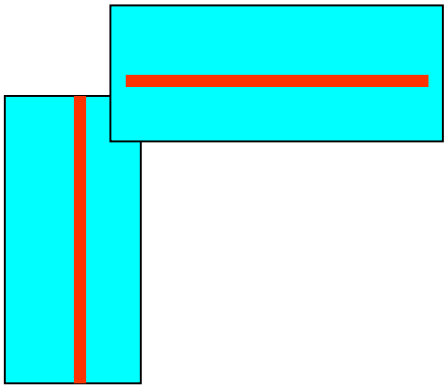
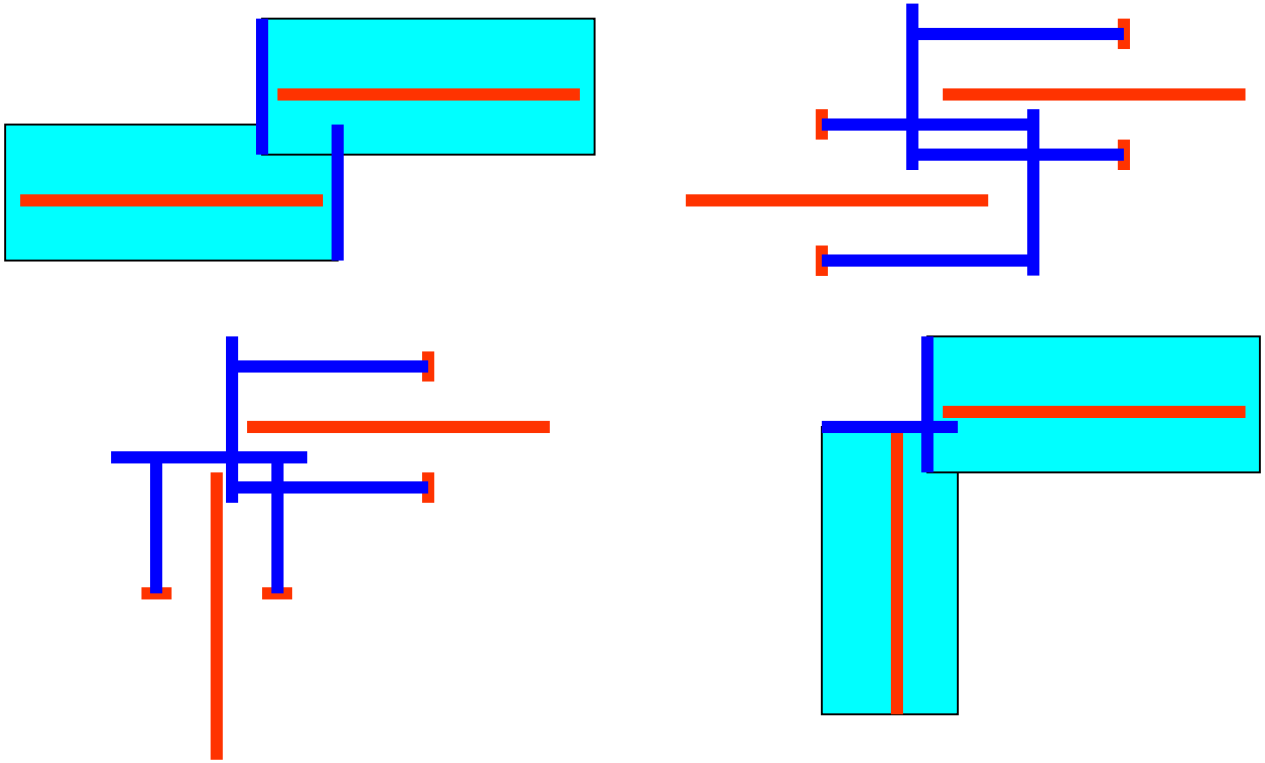
Concatenating building blocks





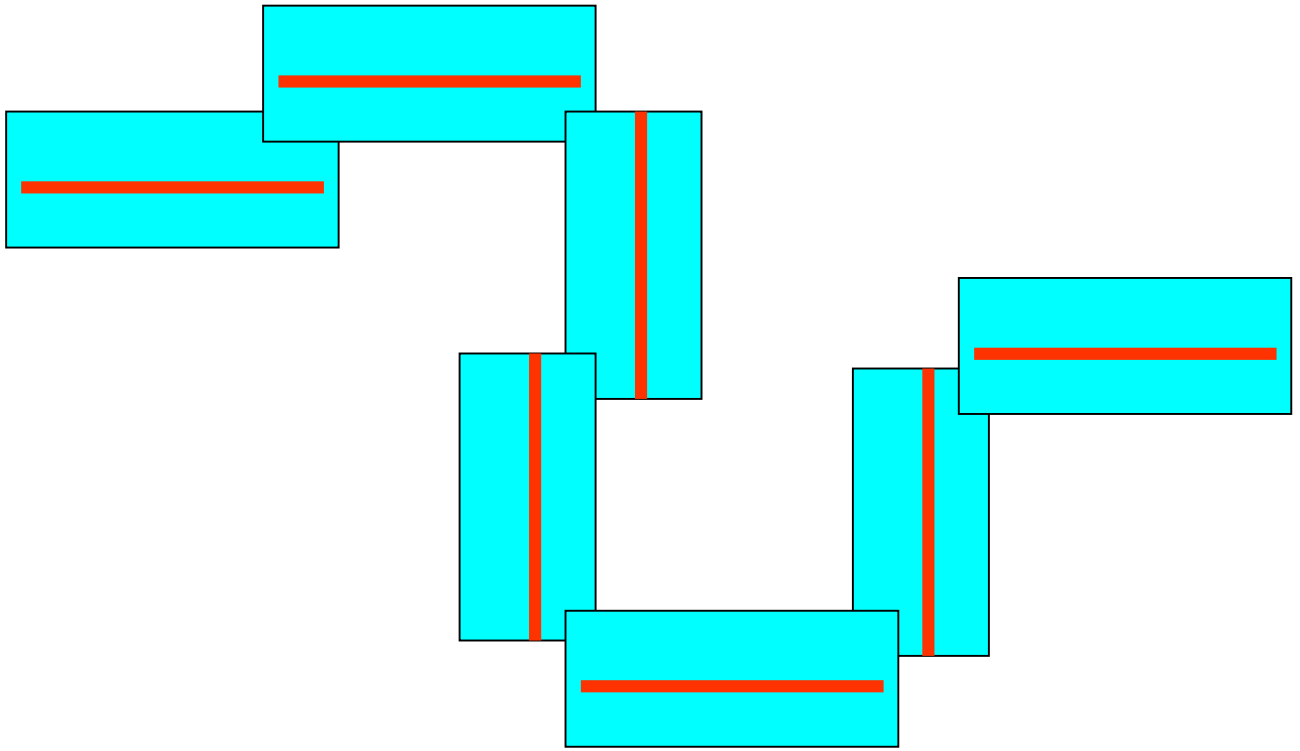


Concatenating building blocks – impossible extensions



Concatenating building blocks

– propagating the *false* value



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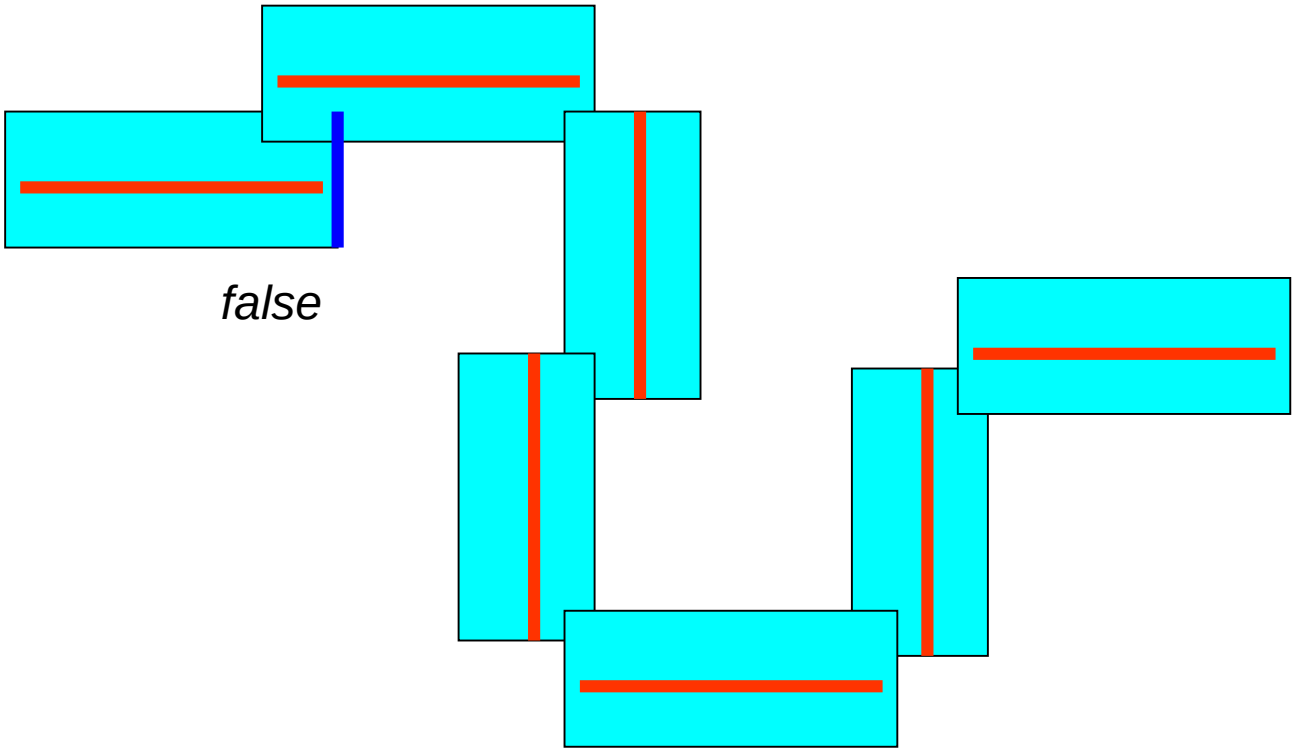
– propagating the *false* value



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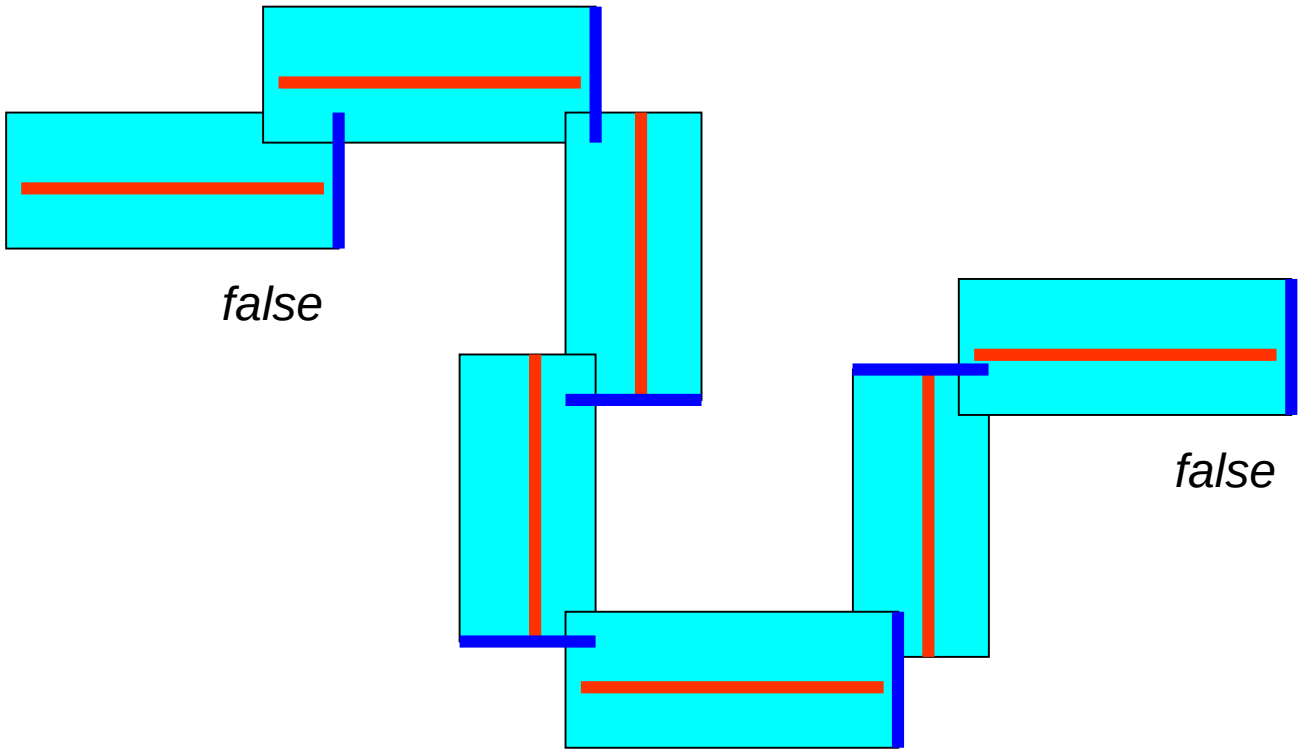
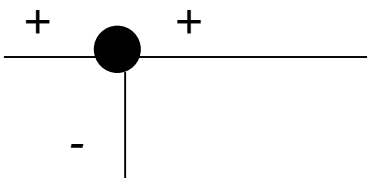
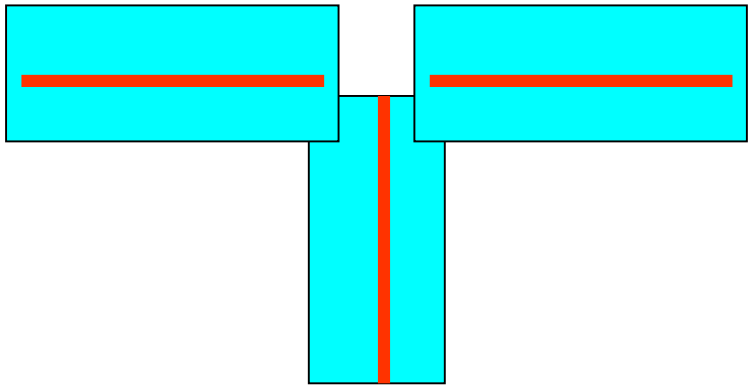


Variable gadget





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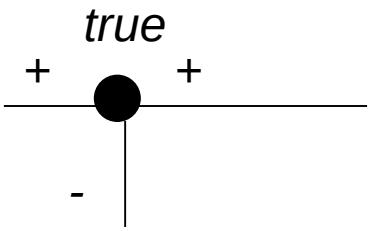

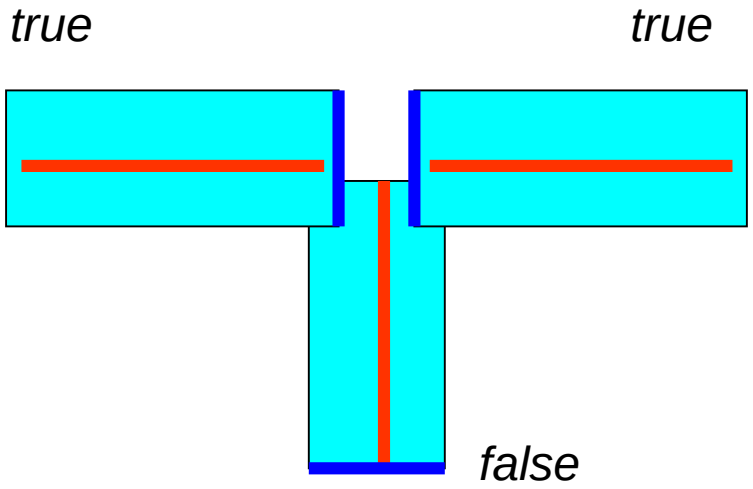

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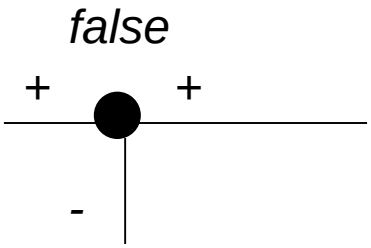

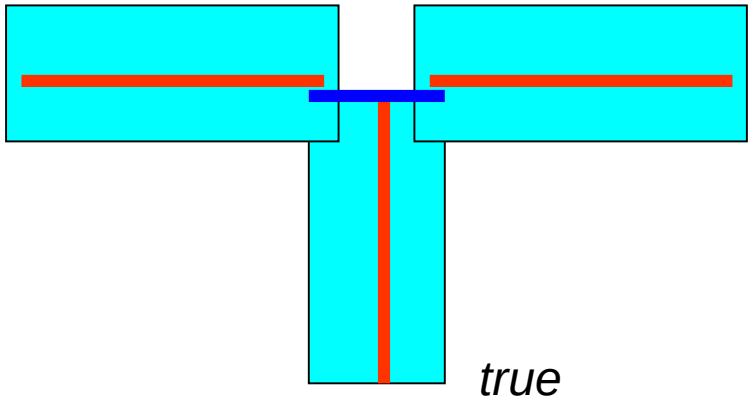

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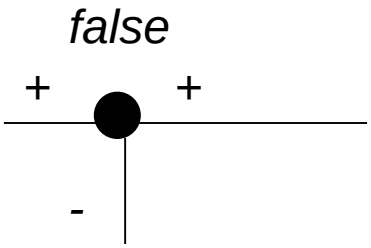
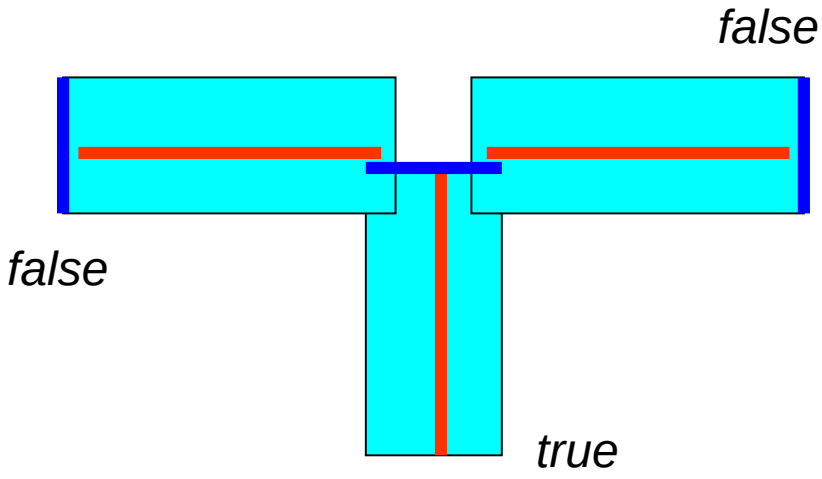
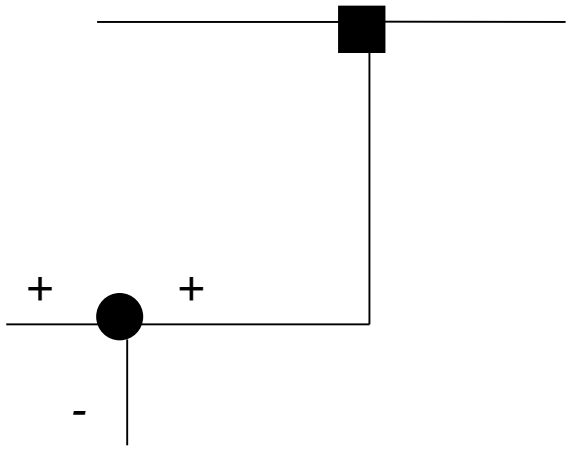
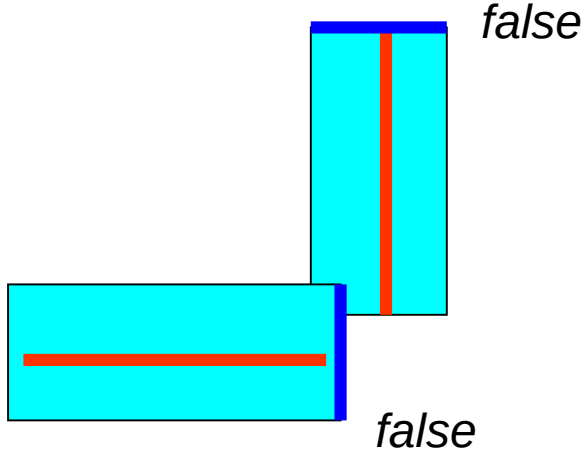


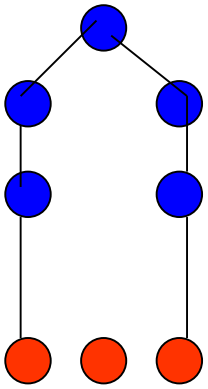


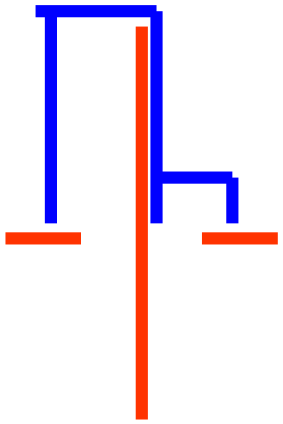
Bend gadget

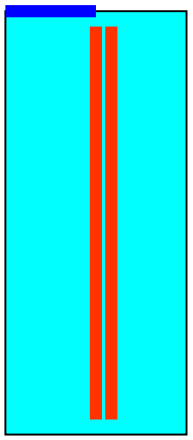




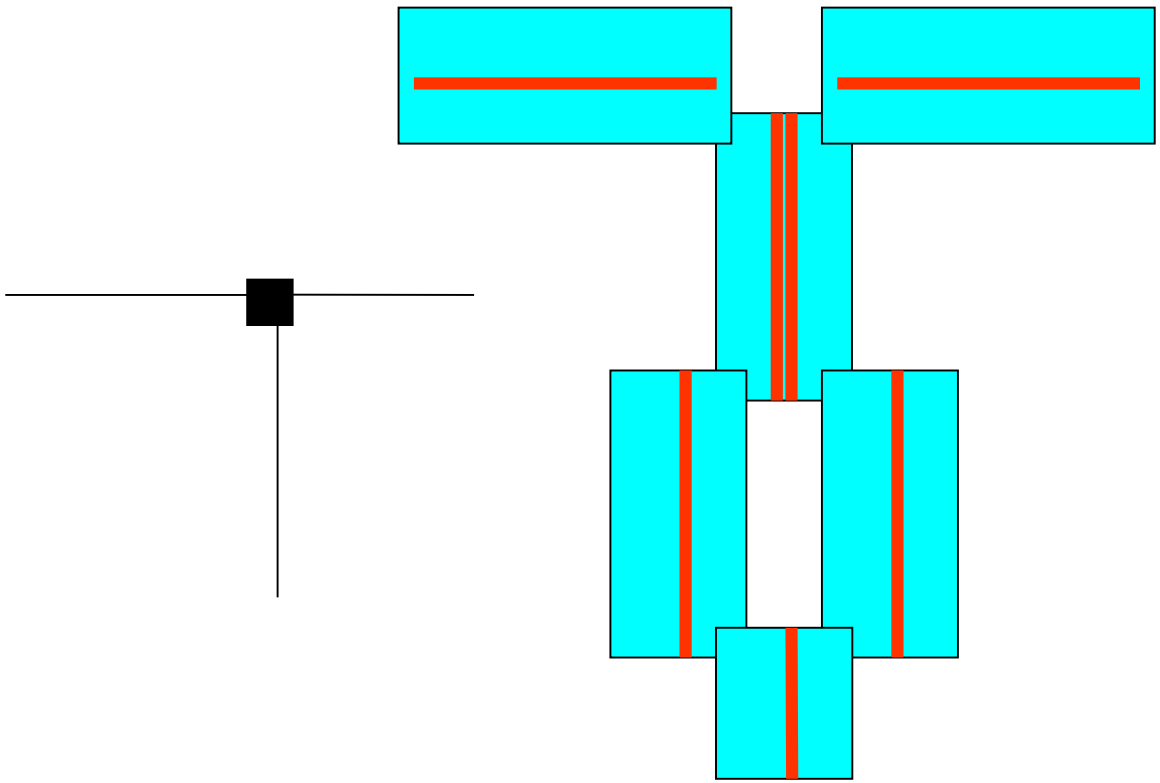
Modified building block







Clause gadget



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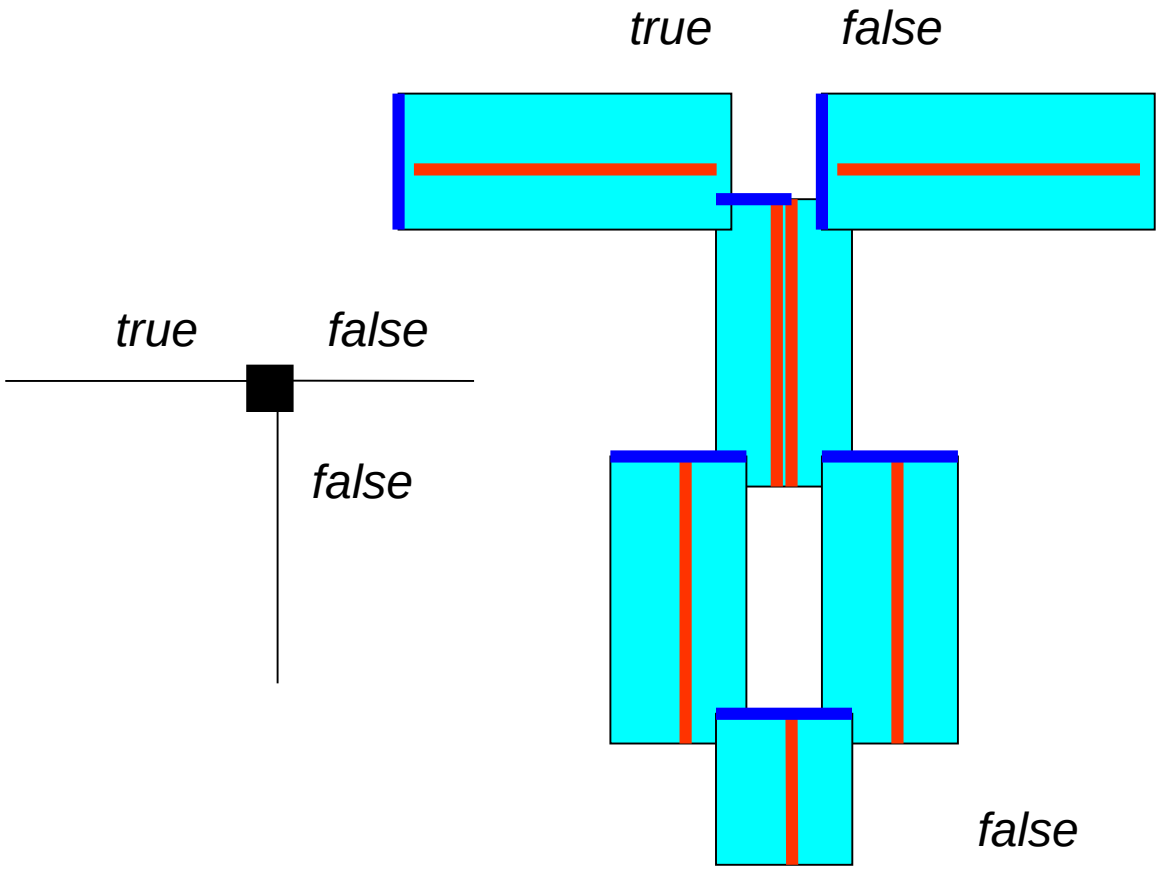

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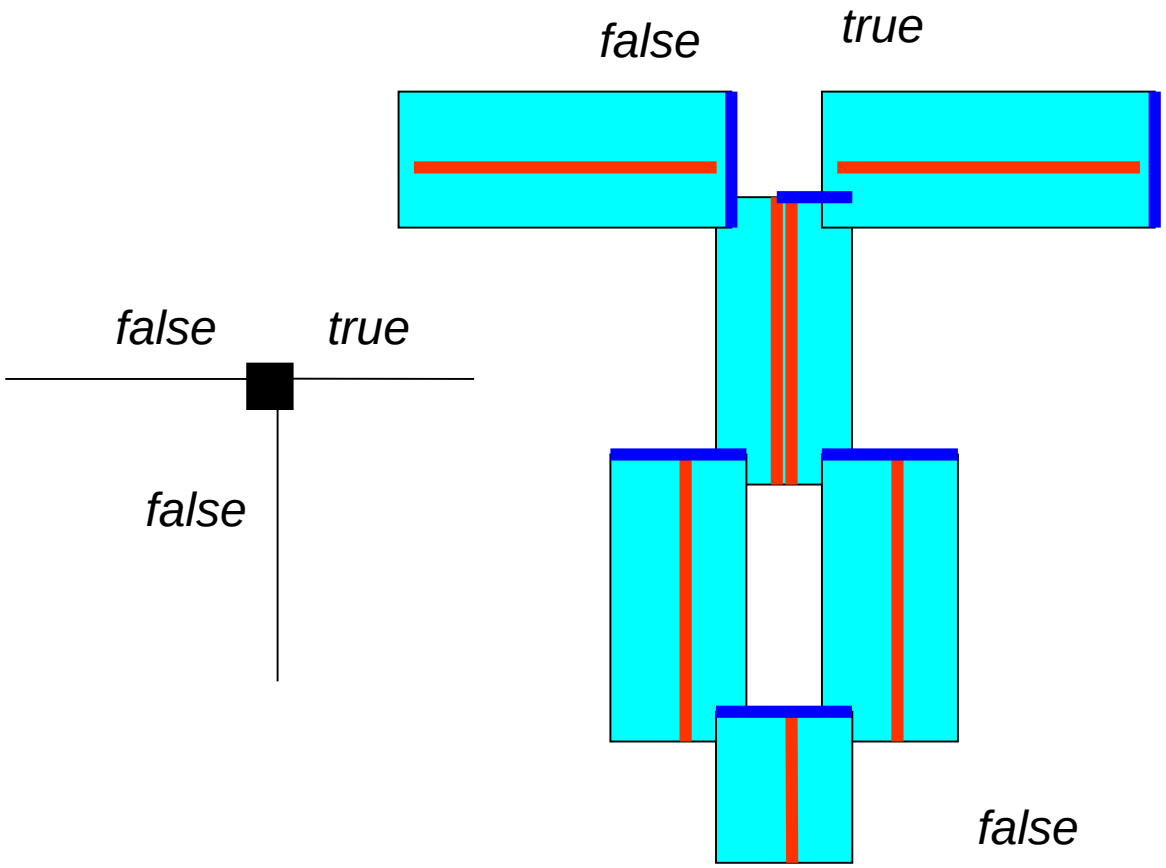

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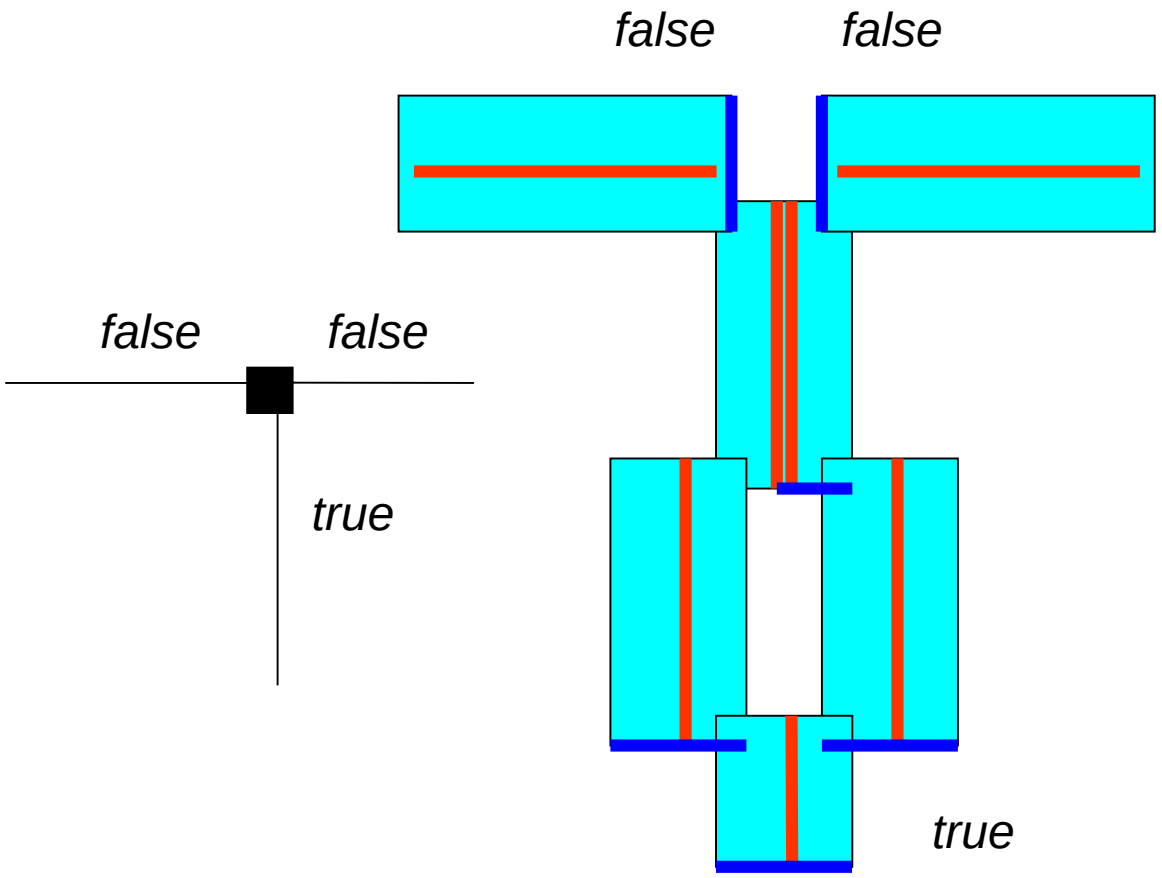
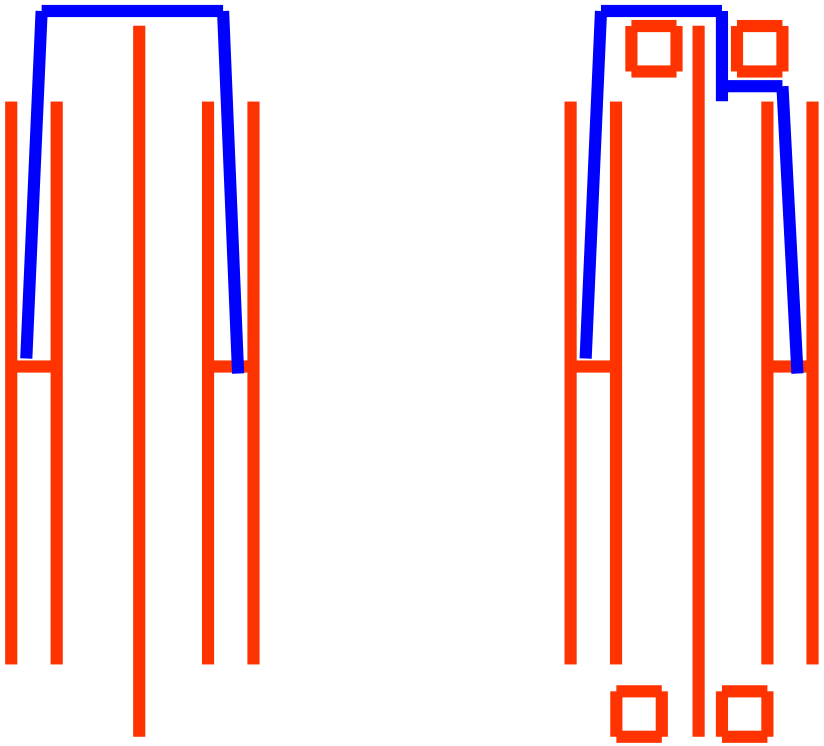
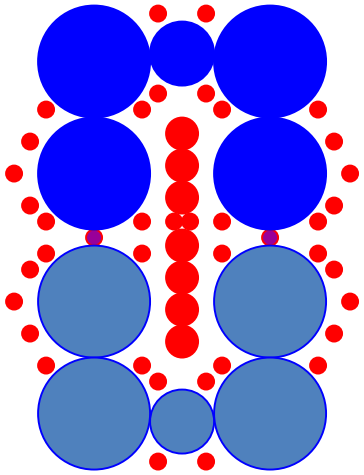
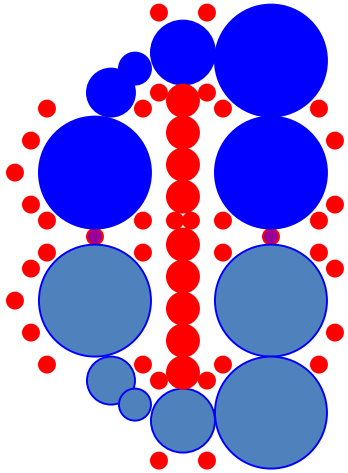


NP-hardness proof
Convex sets, triangles, segments

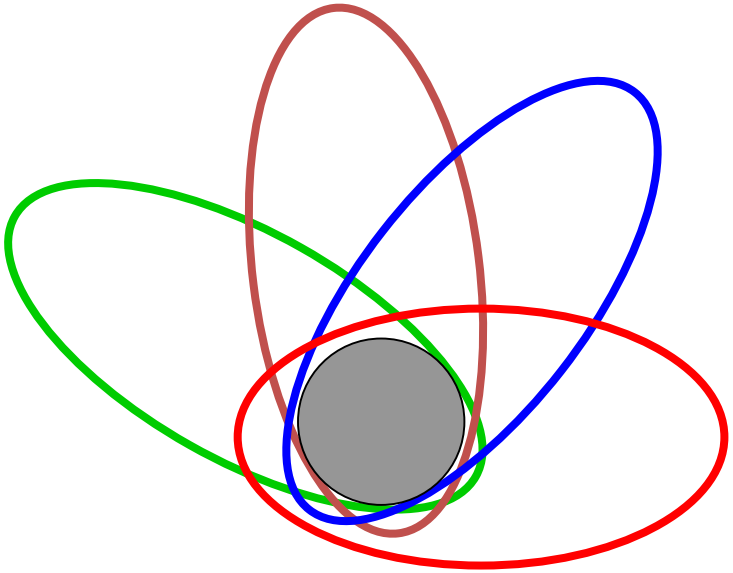


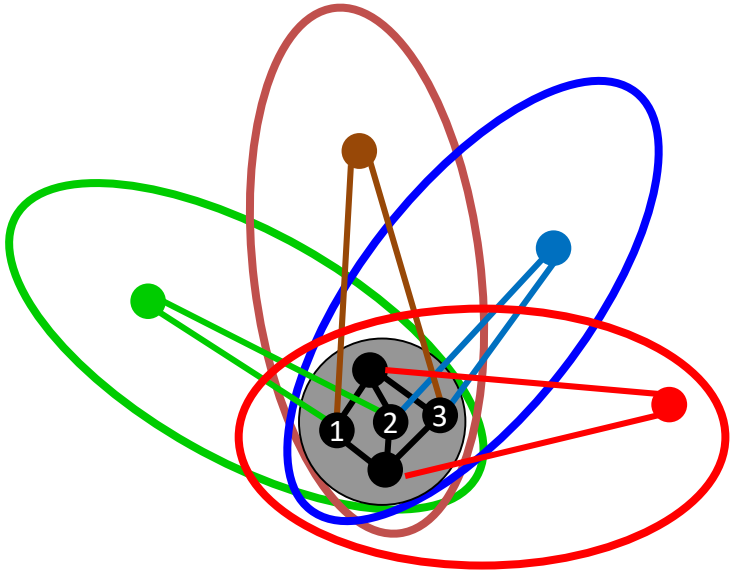
NP-hardness proof
Disks





Simultaneous Contact Representations of Planar Graphs

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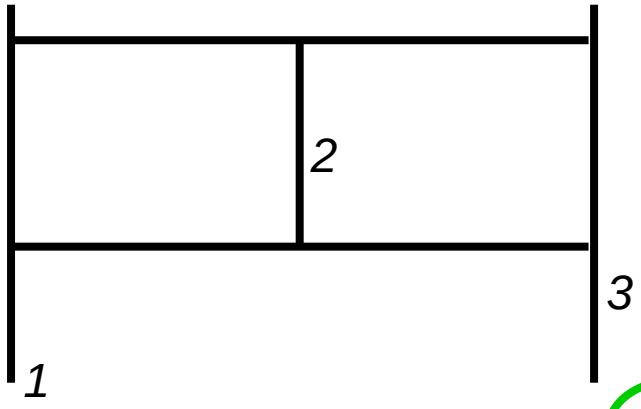
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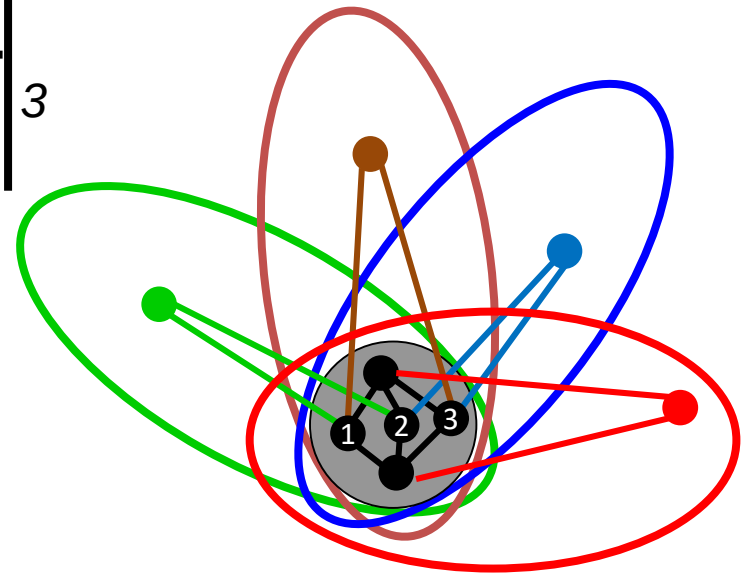
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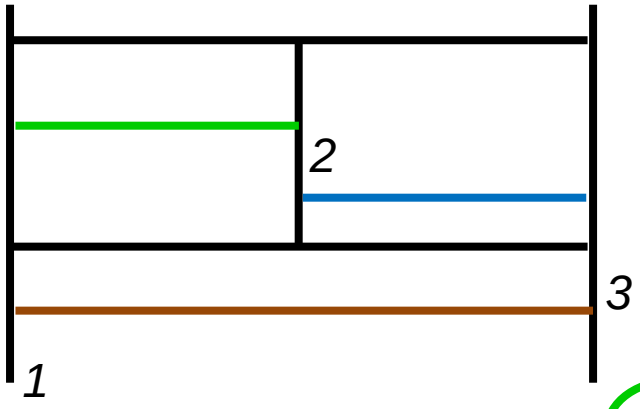
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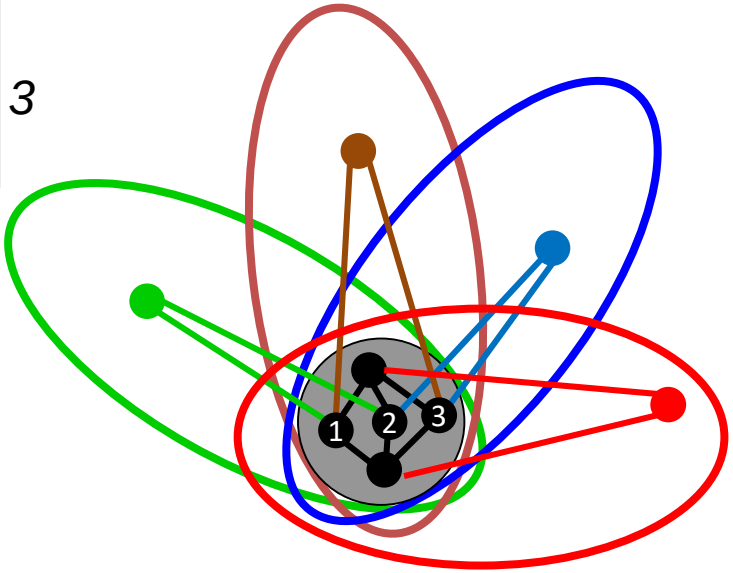
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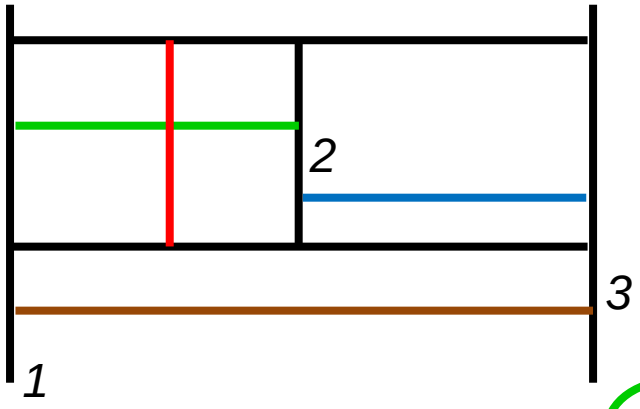
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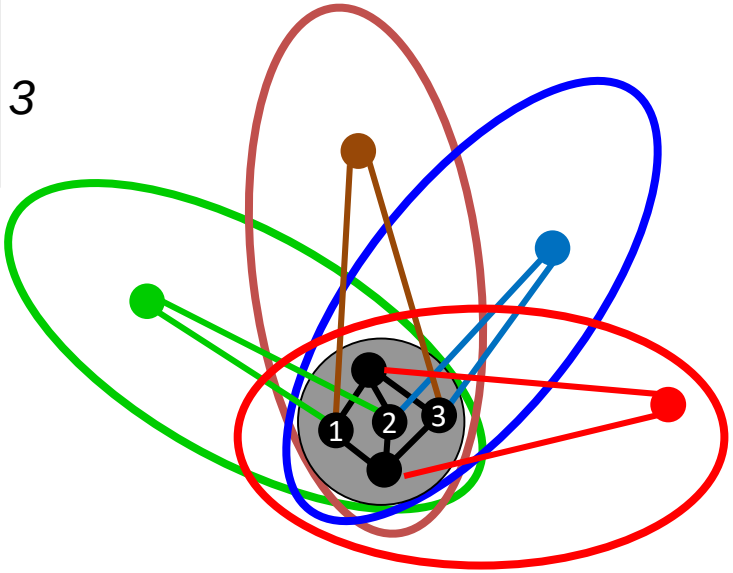
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2
3

Thm (CDKMS 2014): RepExt-GridContact is NP-complete.

Thm: RepExt-GridContact $\propto$ SimRep-3-GridContact.

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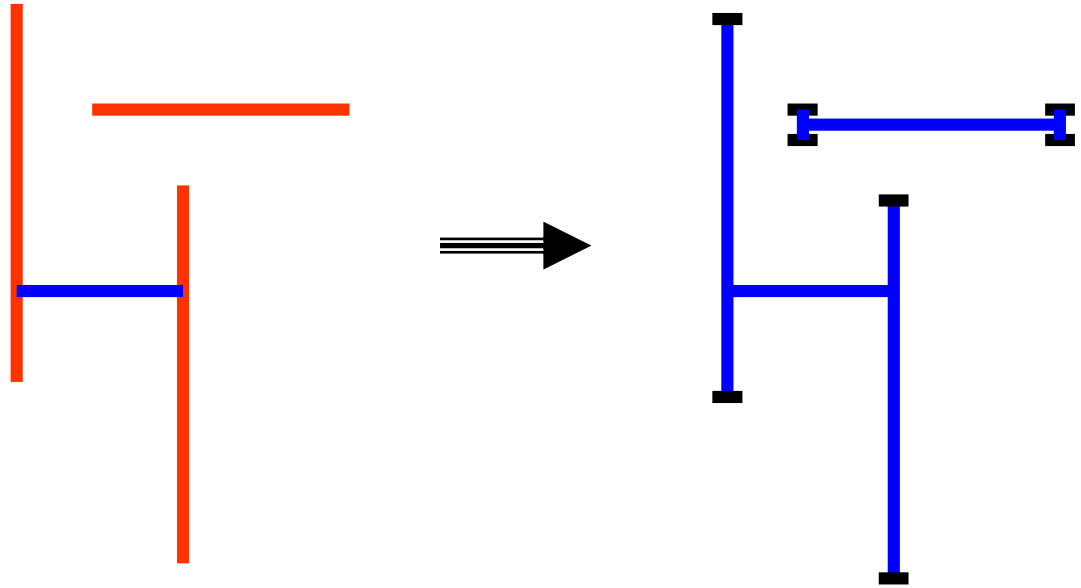

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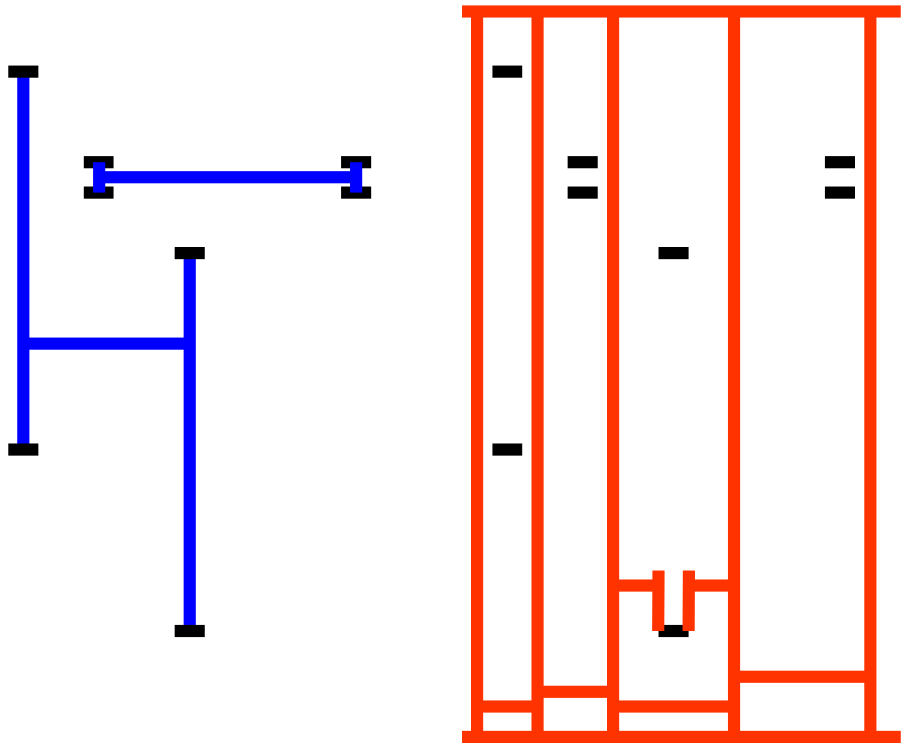

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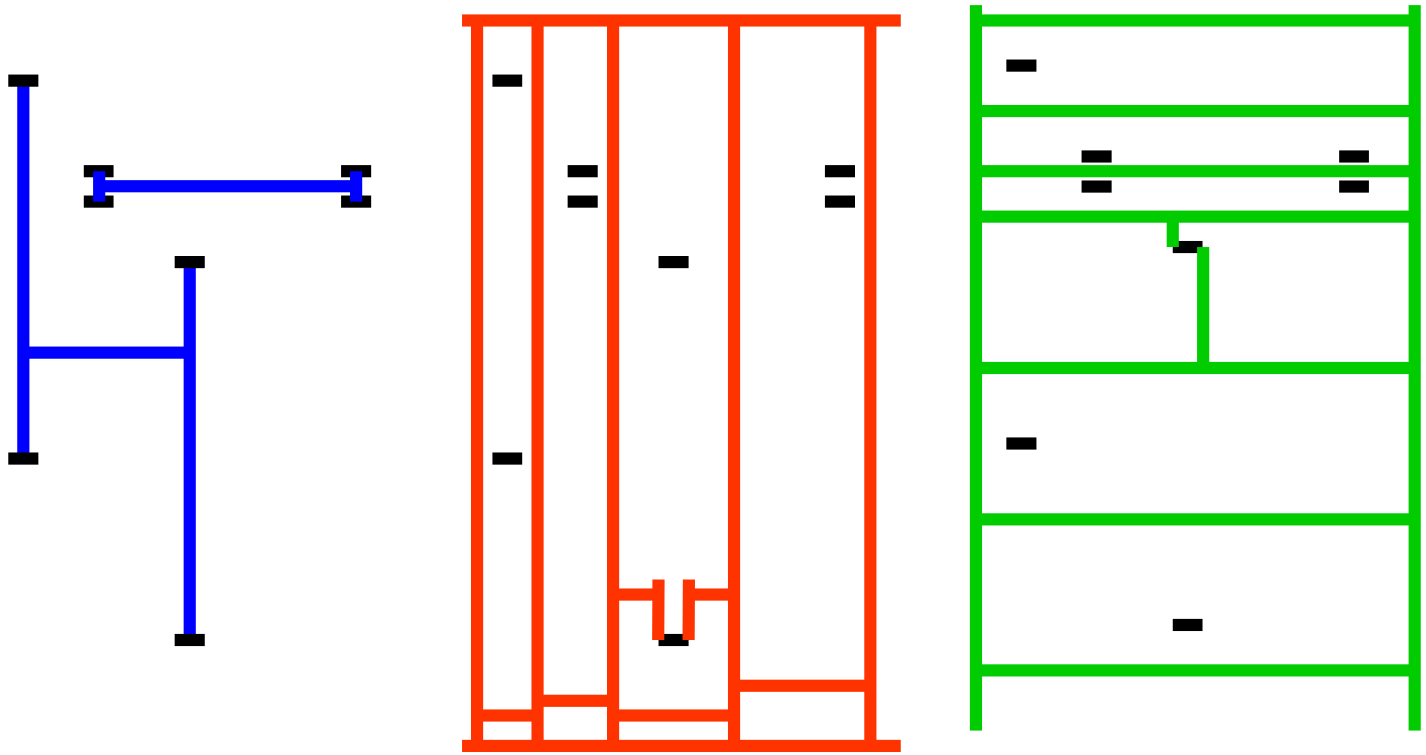

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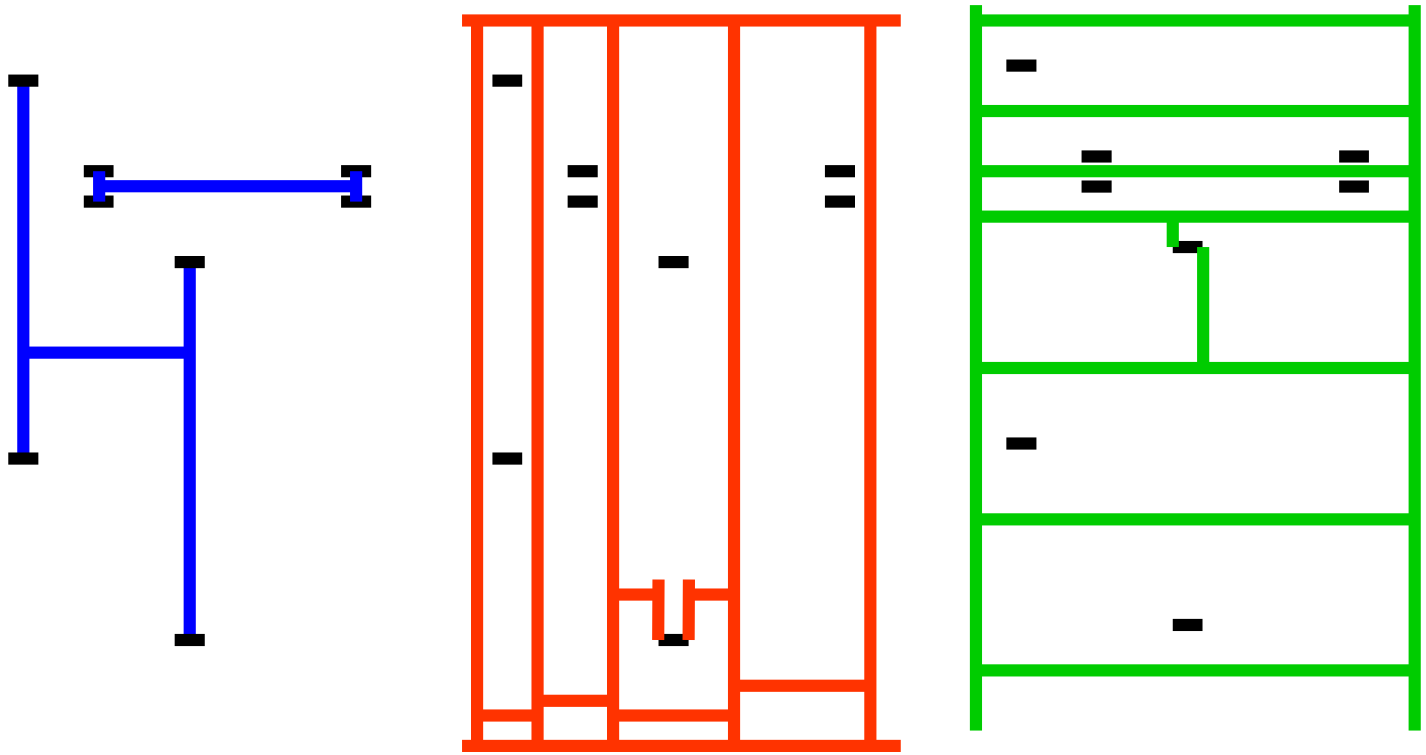
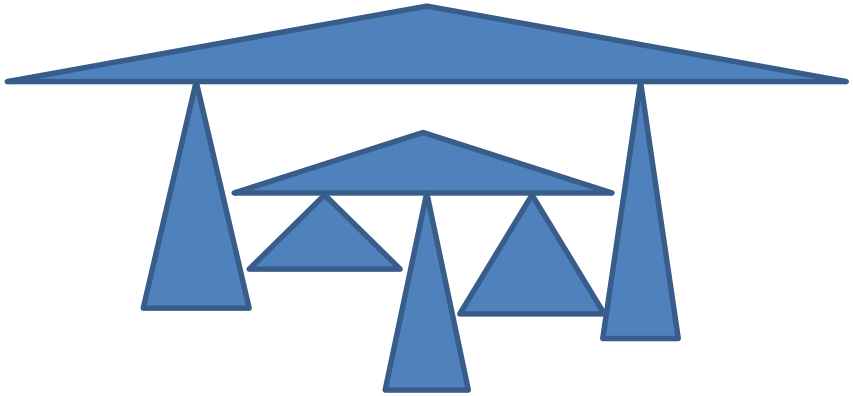


Cor (CDKMS 2016): SimRep-3-GridContact is NP-complete.

Shonan 2020 Open problems:

Complexity of SimRep-2-GridContact?

Complexity of SimRep-k-IsoscelesTrianglesContact?



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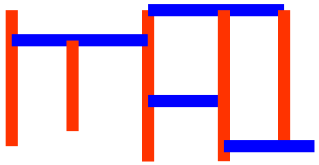
Thm 2 (JK, Reihl – FCT 2025): SimRep-2-GridContact is NP-complete.



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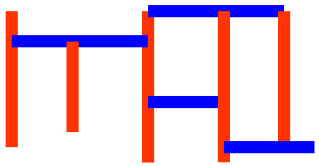
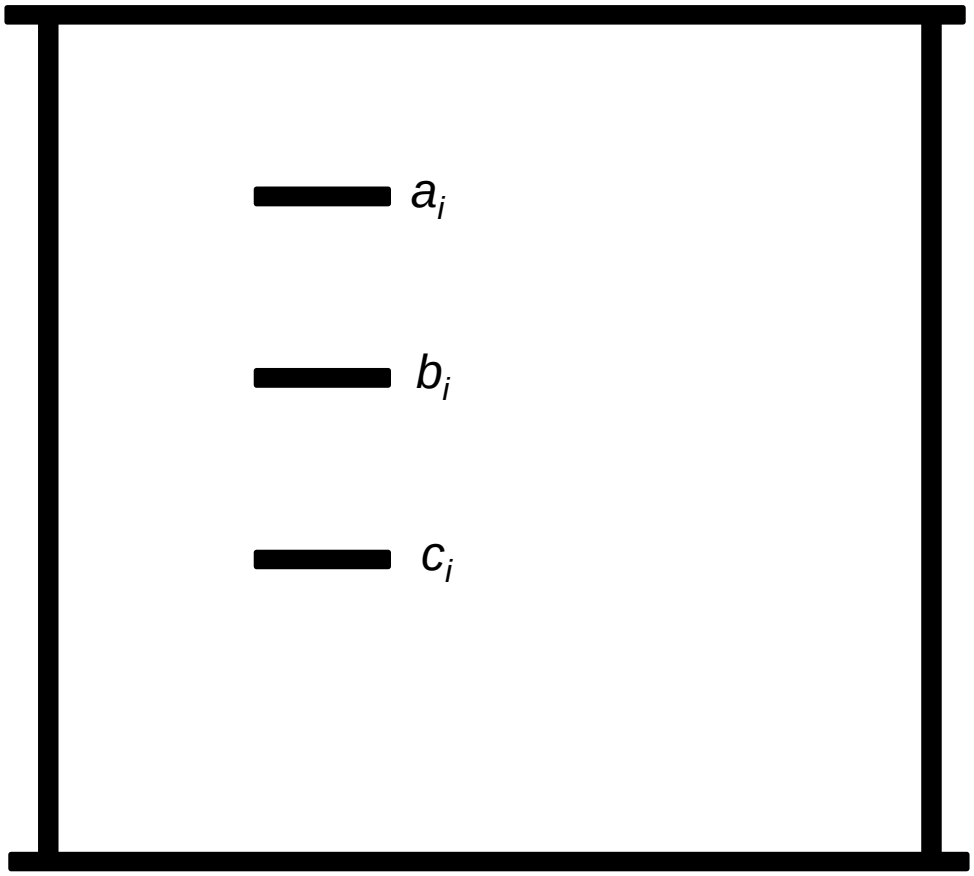
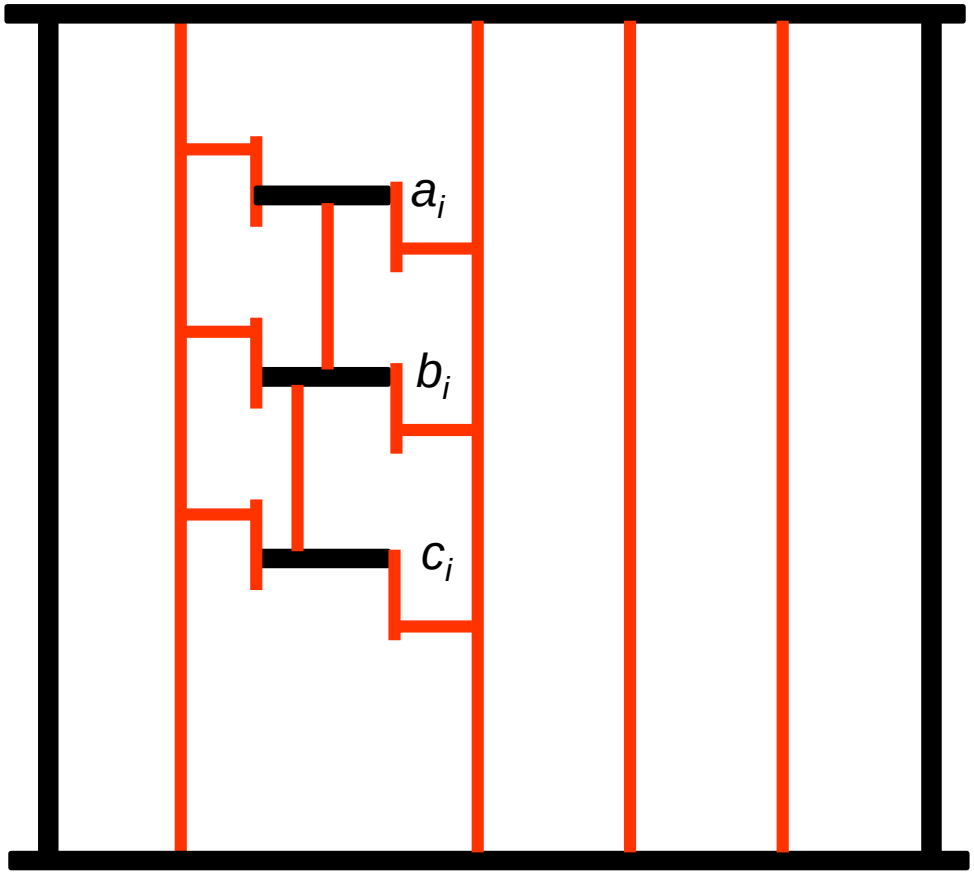
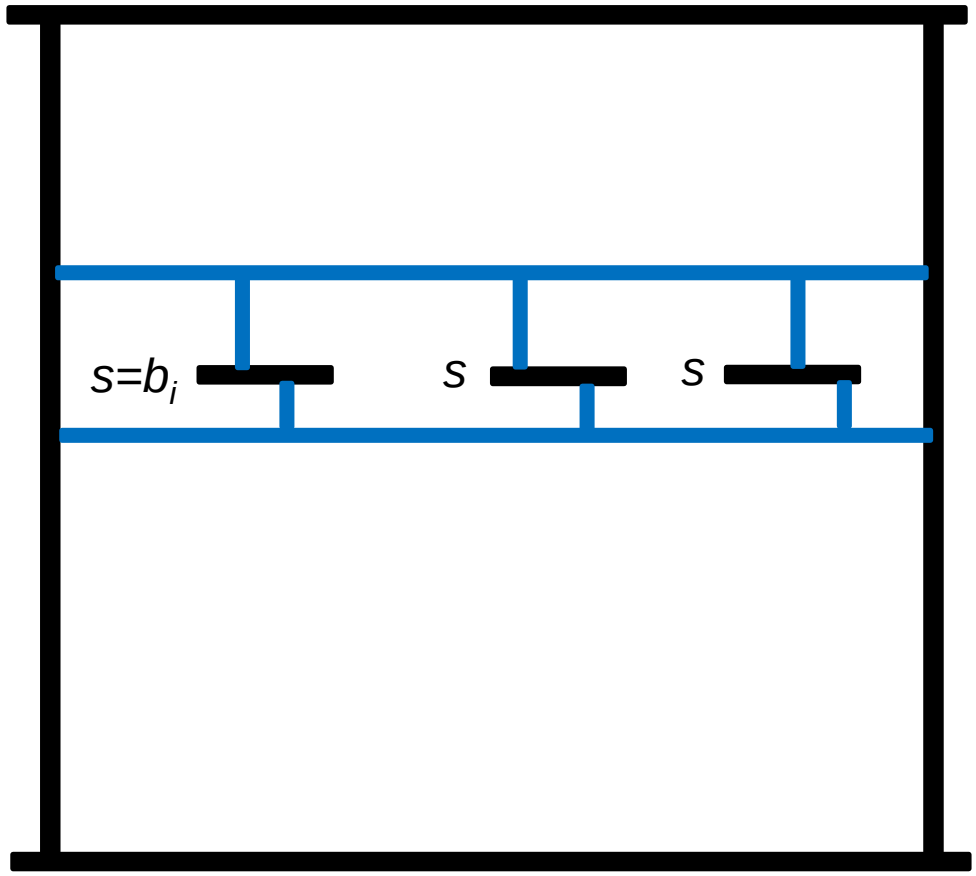


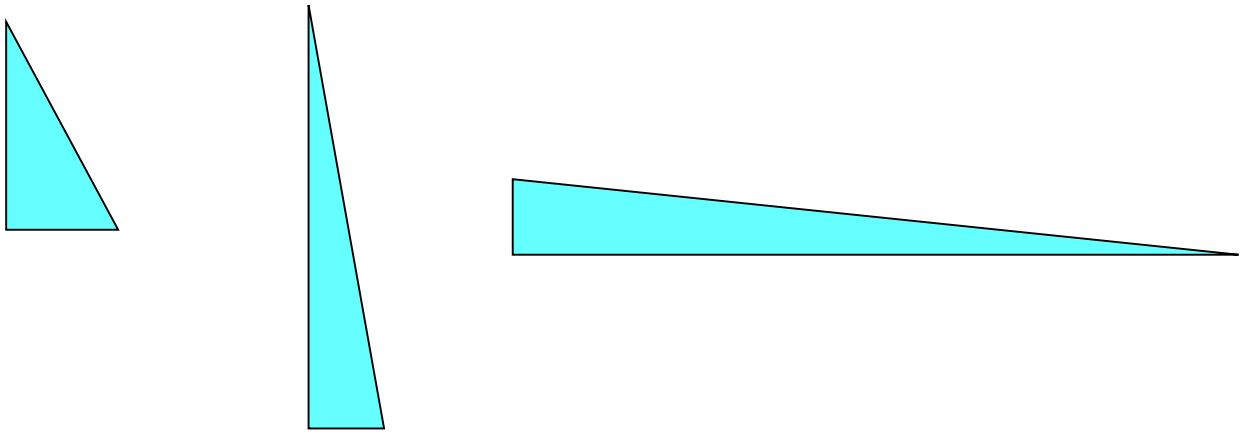
Proof: Reduction from Betweenness
Given triples $T_i=(a_i,b_i,c_i)$ over S, is there a linear ordering < of S such that for every i, b_i is between a_i and c_i (i.e., either $a_i<b_i<c_i$ or $c_i<b_i<a_i$)? – NP-complete by Opatrny (1979)


a_i
b_i
c_i


a_i
b_i
c_i


$s=b_i$
s
s

Definition: A set C of bounded convex sets in the plane is x-y-stretchable it is closed on translations and on stretching in the x and y directions.

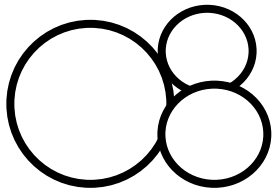


Examples: Axes-aligned rectangles, right-angled triangles, isosceles triangles, segments, ellipses

Thm 2' (JK, Reihl – FCT 2025): Let C be a non-empty x-y-stretchable set of convex sets in the plane. Then SimRep-2 C-Contact is NP-complete.

Open problem

SimRep of disk-contact graphs?



Open problem

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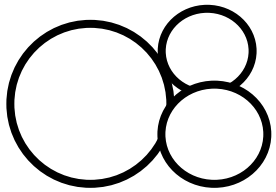

Thank you!