Coalition formation (hedonic) games

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1 Basic definitions

Formally, a *hedonic game* is a pair $(N, (\succcurlyeq_i)_{i \in N})$ of a finite set N of players (or agents), and, for each player $i \in N$ a complete (total) and transitive preference relation $\succcurlyeq_i$ over the set $\{S \subseteq N : i \in S\}$ of coalitions that player i belongs to. A *coalition* is a subset $S \subseteq N$ of the set of players. The coalition N is typically called the *grand coalition*.

A *coalition structure* π is a partition of N. Thus, every player $i \in N$ belongs to a unique coalition $\pi(i)$ in π.

2 Introductory examples

- Persona non grata,
- Stable marriage problem (see Figure 1),
- Stable roommate problem.

3 Notions of stability

- Core-stable (C),
- Nash-stable (NS),
- individually stable (IS),
- individually rational (IR),
- et cetera, see the Figure 2 below.

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4 Exercises

Exercise 1. Show that if a partition is NS, then it is IS.

Exercise 2. Show that if a partition is IS, then it is IR.

Exercise 3. Show that if a partition is C, then it is IR.

Exercise 4. For the Persona non grata game decide if the parition $P = \{\{1, 2\}, \{3\}\}$ is core-stable, Nash-stable or individually stable.

5 Restrictions on preferences relation

- Additively separable games (see Figure 3 for example),
- fractional games,
- hedonic coalition nets,
- anonymous hedonic games,
- individually rational lists of coalitions,
- graphical games [4].

6 Conclusion

For more information on hedonic games, see surveys [1, 3]

References

[1] Aziz, Haris, and Rahul Savani. "Hedonic games." Handbook of Computational Social Choice (2016): 136-139.

[2] Ballester, Coralio. "NP-completeness in hedonic games." Games and Economic Behavior 49.1 (2004): 1-30.

[3] Hajduková, Jana. "Coalition formation games: A survey." International Game Theory Review 8.04 (2006): 613-641.

[4] Peters, Dominik. "Graphical Hedonic Games of Bounded Treewidth." AAAI. 2016.

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function stableMatching {
    Initialize all m in M and w in W to be free
    while exists free man m who still has a woman w to propose to {
       w = first woman on m's list to whom m has not yet proposed
       if w is free
         (m, w) become engaged
       else some pair (m', w) already exists
         if w prefers m to m'
            m' becomes free
           (m, w) become engaged
         else
           (m', w) remain engaged
    }
}
```

Figure 1: Algorithm of Gale & Shapley. (Source: Wikipedia)

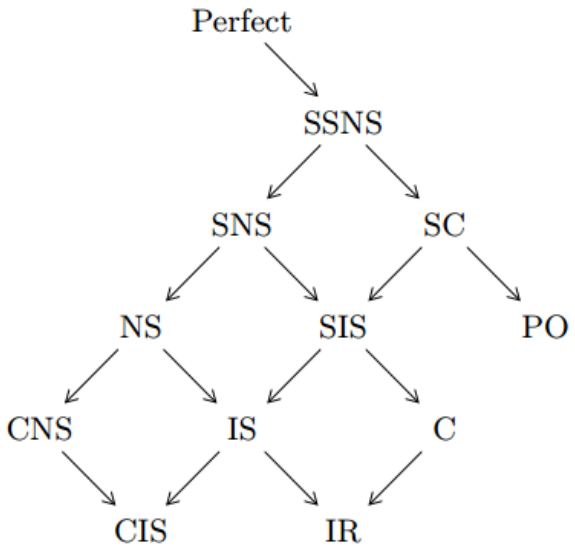

Figure 2: Hasse diagram of relations between the different notions of stability. [1]

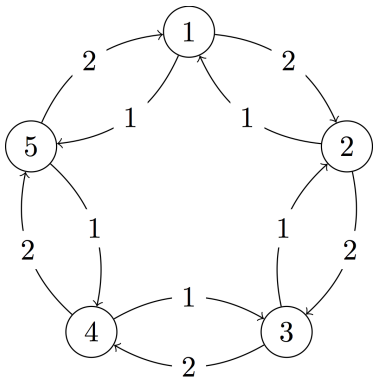

Figure 3: This digraph describes an additively separable hedonic game whose core is empty. It has five players (displayed as circled vertices). Any two players not connected by an arc have valuation -1000 for each other. (Source: Wikipedia)