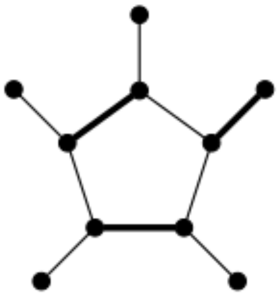
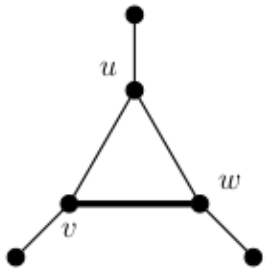
Exercise 9: LP & CO

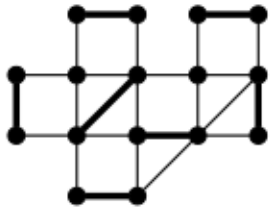
1. Apply Edmonds' algorithm on each of the graphs below starting with the matching shown in bold.



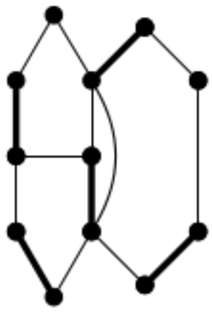
G_1



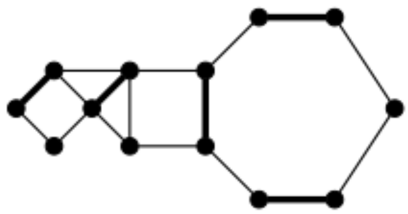
G_2



G_3



G_4



G_5

2. suppose we are given an algorithm A that checks whether a given system of linear inequalities is feasible or not in time polynomial in the number of inequalities. Describe how we can solve aribitrary linear programs in time polynomial in the number of constraints using A.

3. Let $M = (E, \mathcal{I})$ be a matroid. An independent set $I \in \mathcal{I}$ is said to be maximal if for all $x \in E \setminus I$ we have that $I \cup \{x\} \notin \mathcal{I}$. An independent set of largest possible cardinality is called maximum. Prove that every maximal independent set of a matroid is also maximum.

4. Prove that $\left([n], \bigcup_{i=0}^{k} \binom{[n]}{i}\right)$ is a matroid for each $0 \leqslant k \leqslant n$, where $\binom{X}{i}$ is the set of all subsets of X that have cardinality i.

5. Let $M_1 = (E_1, \mathcal{I}_1)$ and $M_2 = (E_2, \mathcal{I}_2)$ be two matroids with $E_1 \cap E_2 = \emptyset$. Let $E = E_1 \cup E_2$ and let $\mathcal{I} = \{x \cup y \mid x \in \mathcal{I}_1, y \in \mathcal{I}_2\}$. Prove that $(E, \mathcal{I})$ is a matroid.

6. Let $G = (V, E)$ be a graph and let $\mathcal{I} = \{F \subseteq E \mid F \text{ is a matching in } G\}$. Give an example where $(E, \mathcal{I})$ is not a matroid.

7. Let $(E, \mathcal{I})$ be an independence system and let $\mathcal{B}$ be the set of all inclusion-wise maximal elements in $\mathcal{I}$. Suppose that $\mathcal{B}$ satisfies the following. For all distinct $X, Y \in \mathcal{B}$ and for all $x \in X$ it holds that there exists $y \in Y$ such that $X \setminus \{x\} \cup \{y\} \in \mathcal{B}$. Prove that for all distinct $X, Y \in \mathcal{B}$ we have $|X| = |Y|$.