*1 Write the following LP in the equational form and specify its data in matrix/vector form:

 $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4$ free, x_5 free

2. Suppose we want to solve the linear optimization problem without inequalities

$$\min \mathbf{c}^{\mathsf{T}} \mathbf{x} \text{ s.t. } \mathbf{A} \mathbf{x} = \mathbf{b},$$

where **A** is some $m \times n$ matrix. Show:

- (a) If there exist $\mathbf{x}_1 \neq \mathbf{x}_2$ with $\mathbf{A}\mathbf{x}_1 = \mathbf{A}\mathbf{x}_2 = \mathbf{b}$ and $\mathbf{c}^{\mathsf{T}}\mathbf{x}_2 > \mathbf{c}^{\mathsf{T}}\mathbf{x}_1$, then the minimum is not bounded from below.
- (b) If there exists $\mathbf{x} \in \mathbb{R}^n$ with $\mathbf{A}\mathbf{x} = \mathbf{b}$ and the minimum is bounded from below, then $\mathbf{c}^{\mathsf{T}}\mathbf{x}$ is a constant for all solutions \mathbf{x} to $\mathbf{A}\mathbf{x} = \mathbf{b}$.
- 3. Consider the following LP in equational form:

- (a) How many bases does the above LP have at most? Find the exact number by enumerating all possibilities. How many of them are feasible bases?
- (b) Consider the point $\mathbf{x} \in \mathbb{R}^5$ where $\mathbf{x} = (0, 4, 0, 5, 0)$.
 - i. If \mathbf{x} feasible for the LP?
 - ii. Is it a basic feasible solution? If so, how many different bases define this point?
- 4. Write the problem min{max{ $\mathbf{c}^{\mathsf{T}}\mathbf{x} \mathbf{c}_0, \mathbf{d}^{\mathsf{T}}\mathbf{x} \mathbf{d}_0$ } s.t. $\mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$ as a linear program.
- 5. Consider the linear programming problem

$$\max x_1 + x_2$$
 s.t. $sx_1 + x_2 \leq t, x_1 \geq 0, x_2 \geq 0$.

Find values for s and t such that this linear program has (a) a finite optimum solution, (b) no feasible solution, and (c) an unbounded optimum.

6. Prove or disprove: If the linear program max $\mathbf{c}^{\mathsf{T}}\mathbf{x}$ s.t. $\mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$ is unbounded, then the linear programming problem max x_k s.t. $\mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$ is unbounded for some k.