Discrete Math
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Quiz 9, Points: 8, Time: 10min
Date: 10.12.2025

Name: _____

Problem 1. *Prove that a graph is connected if and only if it has a spanning tree.*

Solution. If G contains a spanning tree T then any two vertices u, v are connected by a unique path in T. Since T is a subgraph of G, this path also exists in G. So G is connected.

If G is connected then either it is a tree, in which case it is itself a spanning tree. If G is not a tree then it is not edge-minimally connected. So there exists an edge whose removal leaves the graph connected. Inductively, this graph contains a spanning tree. Since subgraph of a subgraph is also a subgraph, G contains a spanning tree. $\square$