

Name: _____

Problem 1. Define the line graph $L(G)$ of a graph and prove that the number of edges in $L(G)$ equals $\sum_{v \in V(G)} \binom{d_G(v)}{2}$ where $d_G(v)$ is the degree of v in G .

Solution. For any vertex v all the edges incident to v are pairwise connected by an edge in $L(G)$. This contributes $\binom{d_G(v)}{2}$ edges to the total number of edges in $L(G)$. Furthermore, every edge $\{e, e'\}$ in $L(G)$ is contributed this was from the unique vertex in $e \cap e'$. The total count follows by summing the contribution from each vertex. \square