

Name: _____

Problem 1. Define "maximum element" and "maximal element" of a poset.

Solution. Let $P = (X, \preceq)$ be a poset.

An element $a \in X$ is called a maximum element if $\forall x \in X : x \preceq a$.

An element $a \in X$ is called a maximal element if $\nexists x \in X : x \neq a \wedge a \preceq x$. $\square$

Problem 2. Prove that a maximum element of a poset – if it exists – is also a maximal element.

Solution. Let $P = (X, \preceq)$ be a poset and let $a \in X$ be a maximum element. Suppose a is not a maximal element. This means that $\exists x \in X : x \neq a \wedge a \preceq x$. Since a is maximum we have $x \preceq a$. But then by antisymmetry $x = a$, a contradiction. $\square$