Discrete Math
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Quiz 3, Points: 8, Time: 10min
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Name: ____________________________

Problem 1. *Consider the relation R defined on $\mathbb{Z}$ as follows: $(x, y) \in R \iff x + y$ is even.*

1. *Prove that R is an equivalence relation.*
2. *What are the equivalence classes of R?*

Solution.

1. Reflexivity: For all $x \in \mathbb{Z}, x + x = 2x$ which is even. So, for all $x \in \mathbb{Z}, (x, x) \in R$.

   Symmetry: Let $(x, y) \in R$. Then $x + y$ is even. So $y + x$ is even. So $(y, x) \in R$.

   Transitivity: Let $(x, y), (y, z) \in R$.

   If x is odd, then since $x + y$ is even, y is odd. Then, since $y + z$ is even z is odd. Since x, z are both odd, $x + z$ is even.

   If x is even, then since $x + y$ is even, y is also even. Then, since $y + z$ is even, z is also even. Since both x, z is even, $x + z$ is even.

   So whether x is odd or even, if $(x, y), (y, z) \in R, x + z$ is even, that is, $(x, z) \in R$.

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