Discrete Math
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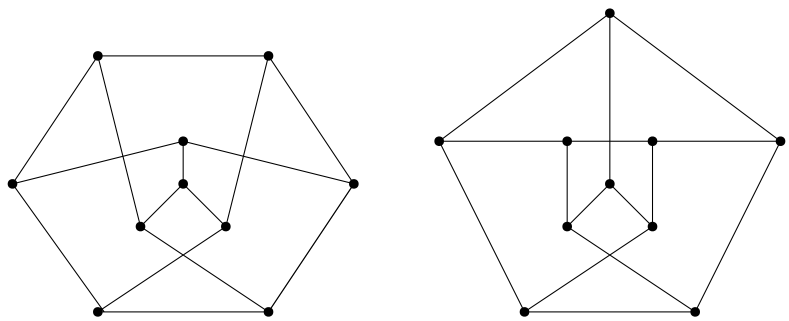
Tutorial 11
HW Due: 03.01.2024

Problem 1. *Decide weather the graphs in pictures are isomorphic.*



Problem 2. *Decide weather sequences* $(1,1,1,2,2,3,4,4,5,5)$ *and* $(1,2,3,4,5,5,6)$ *are degree sequences of a simple graph and try to construct the graph.*

Problem 3. *For a graph* G*, we say that a map* $f : V(G) \to V(G)$ *is an automorphism if it's a graph isomorphism. Find a nontrivial graph whose only automorphism is the identity map and prove that it is such.*

Problem 4. *Let* G, H, I *be three graphs and* $g : G \to H$*,* $f : H \to I$ *isomorphisms. Prove that* $f \circ g : G \to I$ *is an isomorphism as well.*

Problem 5. *Let* $\mathbb{G}$ *be the set of all graphs on finite number of vertices and let* $\leq$ *be a relation on* $\mathbb{G}$ *defined by* $H \leq G$ *iff* G *contains an induced subgraph isomorphic to* H*. Show that* $H \leq G \wedge G \leq H \implies G \simeq H$*.*

Problem 6. *Show that* $\simeq$ *is an equivalence relation on the set of graphs on* n *vertices. Show that* $\simeq$ *has at least* $\frac{2^{\binom{n}{2}}}{n!}$ *equivalence classes.*

Problem 7. *(*) Let* $G(V, E)$ *be a graph. The* excentricity *of a vertex* $V \in V$ *– denoted by* $\mathrm{ex}(v)$ *– is defined to be* $\max_{w \in V} d_G(v, w)$*, where* $d_G(x, y)$ *denotes the length of a shortest path in* G *between* x *and* y*. The* center *of a graph* G *– denoted by* $\mathrm{CT}(G)$ *– is the set* $\{v \mid \mathrm{ex}(v) \leqslant \mathrm{ex}(w) \forall w \in V\}$.

1. *Compute* $\mathrm{CT}(K_n), \mathrm{CT}(K_{m,n}), \mathrm{CT}(C_n), \mathrm{CT}(P_n)$.
2. *Show that if* G *is a tree than* $|\mathrm{CT}(G)| \leqslant 2$.

Problem 8. *(HW) For a graph* G *on* n *vertices, we define the adjacency matrix* $A(G)$ *to be the matrix which has* n *rows and* n *columns corresponding to the vertices of* G*. We define* $M(G)_{ij} = 1$ *if there is an edge between vertices* i *and* j *and* $M(G)_{ij} = 0$ *otherwise. Show that if* $M(G) = M(H)$ *then* $G \sim H$*.*

Problem 9. *(HW) Describe all automorphisms of* K_n *and* $K_{n,m}$ *where* $n \neq m$*.*