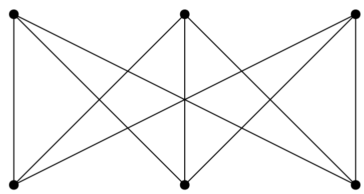
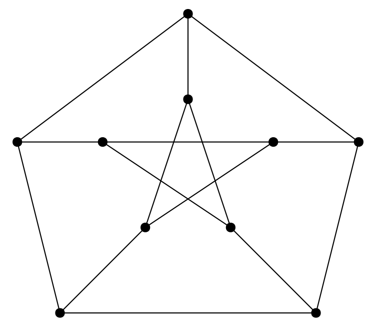
Discrete Math — Tutorial 10
Todor Antić/Hans Raj Tiwary — HW Due: 27.12.2023

Problem 1. *Find the chromatic number of P_n, C_n and K_n for all value of n.*

Problem 2. *Find the chromatic number of the graphs in pictures.*

Problem 3. *We say that a graph G on n vertices is k-degenerate if each* ***induced*** *subgraph H of G contains a vertex of degree at most k. Show that a graph is k-degenerate iff each subgraph contains a vertex of degree at most k.*

Problem 4. *Show that there is no graph G, such that G has 6 vertices and 13 edges and $\chi(G) \leq 3$.*

Problem 5. *Let G be a graph without two disjoint odd cycles. Prove that $\chi(G) \leq 5$.*

Problem 6. *Show that a graph G on n vertices is k-degenerate if and only if admits a linear ordering $v_1 < v_2 < \dots < v_n$ on the vertices such that each v_i forms at most k edges with vertices coming before it in the ordering.*

Problem 7. *(*) We say that a graph G is outerplanar if it can be drawn in the plane without edge crossings and with all vertices on the outer face, A dual graph of a planar graph G is the graph G^* whose vertices correspond to faces of G and two faces are connected by an edge if they share at least one edge.*

1. *Show that every subgraph of an outerplanar graph is outerplanar.*
2. *Prove that the dual of an outerplanar graph is a forest.*
3. *Conclude that every outerplanar graph has a vertex of degree 2.*
4. *Prove that every outerplanar graph is 3-colorable*

Problem 8. *(HW) Let G be a planar, triangle-free graph. Use Euler theorem to prove that G contains a vertex of degree at most three. Then use this to prove that $\chi(G) \leq 4$. You might want to use induction.*

Problem 9. *(HW) Let G be a graph on n vertices. We call an induced subgraph H of G a clique, if it is isomorphic to K_l for some value of l and we call an it an independent set if it is isomorphic to an empty graph. We denote the sizes of the largest clique and independent set of G by $\omega(G)$ and $\alpha(G)$ respectively. With this, show the following:*

1. $\chi(G) \geq \omega(G)$

2. $\chi(G) \geq \frac{n}{\alpha(G)}$