Problem 1. *Draw all non-isomorphic graphs with vertex set* $\{1, 2, 3, 4\}$.

Problem 2. *Draw all non-isomorphic trees on* 6 *vertices.*

Problem 3. *Prove that each graph on* n *vertices with* c *components has at least* $n - c$ *edges.*

Problem 4. *Prove that edges of each eulerian graph can be decomposed into a disjoint union of cycles. Use induction on the number of edges!*

Problem 5. *Prove that a graph and it's complement can't both be disconnected.*

Problem 6. *Suppose that a tree has a vertex of degree* k, *show that it has at least* k *leaves.*

Problem 7. *Graph* G *is* 2*-connected if each two vertices* $u, v \in V(G)$ *are connected by two vertex-disjoint paths in* G. *Diameter of* G *is defined as* $diam(G) = \max_{u,v \in V(G)} d(u, v)$. *where* $d(u, v)$ *is the length of the shortest path between* u *and* v. *With these definitions prove the following:*

1. *Show that in a* 2*-connected graph each vertex is contained in a cylce.*
2. *Show that there is no graph* G *such that both* G *and* $\overline{G}$ *have diameter greater than three. That is, if* G *has diameter at least* 4 *then* $\overline{G}$ *has diameter at most* 3.

Problem 8. *(*) Let* G *be a graph on* $2k$ *vertices in which not three vertices form a triangle. Prove that* G *has at most* k^2 *edges using induction on* k.

Problem 9. *(HW) Prove that every* k*-regular graph contains* P_k *as a subgraph.*

Problem 10. *(HW) A* rooted binary tree *is a tree where one of the vertices is labeled as* root vertex *and each node has zero,one or two child vertices connected to it (see picture below for example). Catalan numbers* C_n *are defined as follows:* $C_0 = 1$ *and* $C_n = \sum_{i=0}^{n} C_{i-1}C_{n-i}$. *Prove that the number of rooted binary trees on* n *vertices is equal to* C_n.
Hint: Consider a tree t ***with*** $n + 1$ ***parent nodes, what can you say about the left and right subtrees of*** t***?***