Discrete Math
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Tutorial 5
HW Due: 22.11.2023

Problem 1. *Assume that we are flipping a fair coin 6 times. What is the probability of the event "There is an even number or heads or there is exactly 3 heads and 3 tails"?*

Problem 2. *A standard deck of 52 cards is dealt out to 4 players so that each player gets 4 cards. What is the probability that none of the 4 players have cards of all four suits? You do not need to simplify the binomial symbols.*

Problem 3. *Suppose $m > 1$ begonias and $n > 1$ fuchsias are randomly arranged on a window sill. All orderings of the $m+n$ flowers are equally likely. What is the probability that to the right of the leftmost begonia there is another begonia?*

Problem 4. *A nonnegative integer solution of $x_1 + x_2 + x_3 = 11$ is picked uniformly at random. What is the probability that $x_1 \leqslant 3, x_2 \leqslant 4$, and $x_3 \leqslant 6$ in the chosen solution?*

Problem 5. *Let (Ω, P) be a probability space and B an event. Consider a function $P' : 2^B \to [0,1]$ defined as $P'(X) = P(X)/P(B)$. Prove that (B, P') forms a probability space.*

Problem 6. *(*) A set of events $A_1, ..., A_i$ is said to be independent if $P(A_1 \cap A_2 \cap \cdots \cap A_i) = \prod_{j=1}^{i} P(A_j)$. Now take a probability space with 8 elements where each event, i.e., equally likely and in it 4 events A, B, C, D such that each triple of events is independent but all 4 events aren't.*

Problem 7. *Let π be a permutation of the set $\{1, \ldots, n\}$. For $1 \leqslant i \leqslant n$, we call i a* fixed point *of π if $\pi(i) = i$. A permutation is called a* derangement *if it has no fixed points.*

1. *List all derangements of $\{1, 2, 3, 4\}$.*
2. *Compute the number of derangements of $\{1, \ldots, n\}$.*
3. *What is the probability that a permutation of $\{1, \ldots, n\}$, picked uniformly at random, is a derangement as $n \to \infty$?*

Problem 8. *(HW) Let R be a relation over a set X. Consider the relation $\preceq_R$ defined over $X \times X$ as follows: $(a_1, b_1) \preceq_R (a_2, b_2)$ if either $a_1 \neq a_2 \wedge (a_1, a_2) \in R$ or $a_1 = a_2 \wedge (b_1, b_2) \in R$. Prove that $\preceq_R$ is an order over $X \times X$ if and only if R is an order over X.*

Problem 9. *(HW) Let X be a set and let R be a relation over X that is both reflexive and transitive. Define the relation $\sim_R$ over X as follows: $a \sim_R b$ iff $(a, b) \in R \wedge (b, a) \in R$.*

1. *Prove that $\sim_R$ is an equivalence relation over X.*
2. *Let $\mathcal{X}_R$ be the set of equivalence classes of $\sim_R$. Define the relation $\preceq_R$ over $\mathcal{X}_R$ as follows: $A \preceq_R B$ iff $\exists a \in A, b \in B : (a, b) \in R$. Prove that $\preceq_R$ defines an order over $\mathcal{X}_R$.*