Problem 1. *Give an example of a partial order on X where X has 15 elements such that width of your partial order is 3 and length is 5.*

Problem 2. *Suppose that in a bushel of 100 apples there are 20 that have worms in them and 15 that have bruises. Only those apples with neither worms nor bruises can be sold. If there are 10 bruised apples that have worms in them, how many of the 100 apples can be sold?*

Problem 3. *Describe all relations on a set X that define an equivalence relation and a partial ordering at the same time.*

Problem 4. *Answer/prove the following:*

1. *For a given poset $(X, \leq)$, any subset $A \subseteq X$ has at most one supremum and at most one infimum.*

2. *Which element is the supremum/infimum of the empty set.*

3. *Give an example of a poset in which every nonempty subset has an infimum but there are subsets with no supremum.*

Problem 5. *Let R be a relation over a set X, and let $Y \subseteq X$.*

1. *Prove that if R is an ordering over X then $R \cap (Y \times Y)$ is an ordering over Y.*

2. *Prove that if R is an equivalence relation over X then $R \cap (Y \times Y)$ is an equivalence relation over Y.*

Problem 6. *Use inclusion-exclusion principle to determine the number of integer solutions of the equation*

$$x_1 + x_2 + x_3 + x_4 = 20$$

satisfying the constraints $1 \leq x_1 \leq 6; 0 \leq x_2 \leq 7; 4 \leq x_3 \leq 8; 2 \leq x_4 \leq 6$.

Problem 7. *(*) Prove Erdős-Szekeres theorem*[1] *using the Pigeonhole principle.*

Problem 8. *Use Erdős-Szekeres theorem to prove the following statement: In a set P of at least $rs - r - s + 2$ points in the plane there is a polygonal path of either $r - 1$ positive slope edges or $s - 1$ negative slope edges. You can assume that no two points have same x coordinate.*

Problem 9. *Use inclusion-exclusion principle to calculate the number of onto functions from X to Y where $|X| = n$ and $|Y| = m$.*

[1] The version in the lecture was actually only a special case of the actual theorem. For general formulation see this link to the Wikipedia entry.

Problem 10. *(HW) Use inclusion-exclusion principle to determine how many numbers below 100 are divisible by 2,3, or 5?*

Problem 11. *(HW) Let X be a set of size n. How many distinct total orders can be defined over X?*

Problem 12. *(HW) Prove that every partial ordering on a finite sex X has at least one linear extension. For a partial ordering $\leq$, linear extension is any linear order $\preccurlyeq$ such that $x \preccurlyeq y \implies x \leq y$.*

Problem 13. *(HW) Determine that number of permutations of 1, 2, . . . , 8 in which no even integers are in their initial positions.*