Problem 1. *Let m, n, r be natural numbers such that $m \geq n \geq r$. Prove that* $\binom{m}{r} \geq \binom{n}{r}$.

Problem 2. *Find a way to place 8 rooks on a regular 8×8 chessboard in such a way that no two are attacking each other. Is it possible to place 9 rooks in such a way?*

Problem 3. *Consider a regular deck of playing cards. How many ways are there to place 4 cards from the deck on the table? How many ways are there to place 4 cards on the table if the first card needs to be a red queen?*

Problem 4.

1. *Let $X = \{1, 2, 3, 4\}$. How many ordered pairs (A, B) can you find, where $A, B \subseteq X$ and $A \cap B = \emptyset$.*
2. *Let $X = \{1, ..., n\}$ for some natural number n. How many ordered pairs (A, B) can you find, where $A, B \subseteq X$ and $A \cap B = \emptyset$.*

Problem 5. *Give an algebraic and a combinatorial proof of the following:*

1. $\binom{n}{2} + \binom{n+1}{2} = n^2$
2. $\forall k, n \in \mathbb{N},\ k \leq n\ k\binom{n}{k} = n\binom{n-1}{k-1}$.

Problem 6. *Give combinatoiral proof of the following:*

1. $\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i}$ ***Hint: If we have a class of*** m ***girls and*** n ***boys, in how many ways can we choose a council of*** k ***people?***
2. $\sum_{m=k}^{n-k} \binom{m}{k}\binom{n-m}{k} = \binom{n+1}{2k+1}$. ***Hint: How many subsets of size*** $2k+1$ ***does the set*** $\{1, 2, \ldots, n+1\}$ ***have?***

Problem 7. ** Let p be a permutation of $\{1, 2, \ldots, n\}$ and let us write it in one line, for example (3 5 7 1 2 6 9 8). Now mark the increasing segments in this permutation, on our example we get (3 5 7—1 2 6—9—8). Let $f(n, k)$ denote the number of permutations of an n element set with exactly k increasing segments. Prove that $f(n, k) = f(n, n+1-k)$.*

Problem 8. *(HW) Consider the $n \times n$ grid with the bottom left point marked $(0, 0)$ and the top right point marked $(n-1, n-1)$. In how many ways can we reach $(n-1, n-1)$ from $(0, 0)$ by only moving right or up. That is, from a point (i, j) we are only allowed to move either to $(i+1, j)$ (moving right) or to $(i, j+1)$ (moving up).*

Problem 9. *(HW) Let $F_0 = 0, F_1 = 1$ and for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$. Let C_n be the number of ways to write n as a sum of 1's and 2's. For example $C_3 = 3$ since we can write $1+1+1 = 2+1 = 1+2 = 3$. Prove that $C_n = F_{n+1}$.*