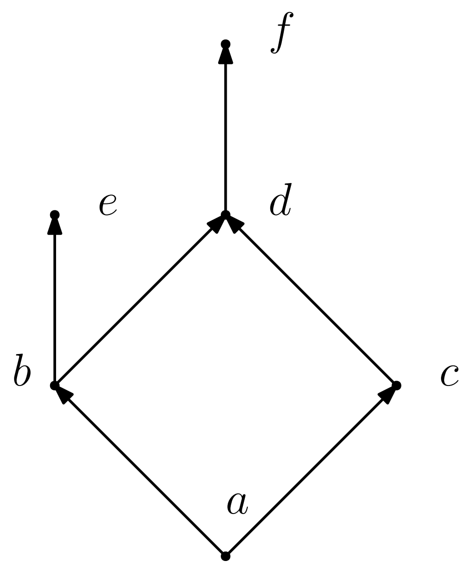
Problem 1. *The picture below can describes a relation R, where $(x, y) \in R$ means that there is a way to get from x to y using the arrows. Write down all the pairs which are in R.*



Problem 2. *Use induction (or strong induction) to prove the following:*

1. $\sum_{i=1}^{n} i(i+1) = \frac{1}{3}n(n+1)(n+2)$.
2. *If* $a_0 = 1, a_1 = 3$ *and* $\forall n \geq 2$, $a_n = 2a_{n-1} - a_{n-2}$ *then* $\forall n \geq 0, a_n = 2n + 1$

Problem 3. *Consider the relations on people "is a brother of", "is a sibling of", "is a parent of", "is married to", "is a descendant of". Which of the properties of reflexivity, symmetry, antisymmetry and transitivity do each of these relations have?*

Problem 4. *Let $f : X \to Y$, $g : Y \to Z$ be surjective functions. Prove that then $g \circ f : X \to Z$ is surjective as well. Find an example where $g \circ f$ is surjective but one of the functions f, g is not.*

Problem 5. *Give a relation R over the set of natural numbers $\mathbb{N}$ such that $R \setminus \{(i, i) | i \in \mathbb{N}\}$ is infinite and R is:*

1. *reflexive, symmetric and transitive.*
2. *reflexive, antisymmetric and transitive.*

Problem 6. *Let $f : X \to Y$ be a function and prove that:*

1. *f is injective if and only if there is a function $g : Y \to X$ such that $g \circ f$ is the identity on X.*

2. *f is surjective if and only if there is a function $g : Y \to X$ such that $f \circ g$ is the identity on Y.*

Problem 7. ** Let R be a relation on a set X. Prove that R is transitive if and only if $R \circ R \subseteq R$*

Problem 8. ** Let R, S be two equivalence relations on a set X. Prove that $R \circ S$ is an equivalence relation if and only if $R \circ S = S \circ R$*

Problem 9. *(HW) Let $f : A \to A$ be a function different from identity such that $f \circ f \circ f = f \circ f$. Prove that f is neither injective nor surjective.*
Hint: Use contradiction and Problem 6!

Problem 10. *(HW) Let R, S be two equivalence relations on a set X. Which of the following are also equivalence relations? Prove your claims!*

1. $R \cup S$
2. $R \cap S$

Problem 11. *(HW) Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, $F_{n+1} = F_n + F_{n-1}$ for $n \geq 1$. Prove the following:*

1. $\sum_{i=1}^{n} F_i = F_{n+2} - 1$
2. $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$

Hint: Your proof should use induction!