Exercise 1: Discrete & Continuous Optimization

1. (a) Show that $(-1, -1)$ is not a local minimum of $f(x, y) = x^2 + y^2 - 4x - 2y + 1$
   (b) Show that $(0, 0)$ is not a local minimum of $g(x, y) = x^3 - 3xy + y^3$
   (c) Show that $(1, 1)$ is not a local minimum of $h(x, y) = e^{-x^2-y^2}(x^2+y^2)$
2. Find the local extrema of the following functions using the second derivative test.
   (a) $f(x, y) = x^2 + y^2 - 4x - 2y + 1$
   (b) $g(x, y) = x^3 - 3xy + y^3$
3. Show that the following sets are convex:
   (a) $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$
   (b) $T = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0\}$
4. Show that the following functions are convex or not convex on their domains:
   (a) $f(x) = x^4 - 2x^2 + 3$ on $\mathbb{R}$
   (b) $g(x, y) = x^2 + y^2$ on $\mathbb{R}^2$
   (c) $h(x, y) = xy$ on $T = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0\}$