Probability and Statistics 1. Exercises 9

1. Check if the estimator of expectation, $\hat{\Theta}$ for a sample of random variables $X_1, \ldots, X_n$ for exponential distribution, is unbiased. Is it consistent? Compute the MSE for this parameter.

2. Consider the estimator for the cdf defined by $\hat{F}_n(k) := \frac{1}{n}\sum_{i=1}^{n} I(X_i \le k)$, where $I(X_i \le k)$ is 1 if $X_i \le k$, and 0 otherwise. Estimate the bias, variance and MSE of $\hat{F}_n(k)$.

3. Prove that $\hat{S}_n^2$ (the corrected sample variance, i.e. the sample variance times $n/(n-1)$) is a consistent estimator. More generally, if an estimator is unbiased and has a vanishing variance (with n going to infinity) then show that the estimator is consistent.

4. Assume a sample of continuous random variables: $X_1, X_2, \ldots, X_n$, where $E[X_i] = \mu$, $\text{Var}[X_i] = \sigma^2 > 0$. Consider the following estimators: $\hat{\mu}_{1,n} = X_n$, $\hat{\mu}_{2,n} = \frac{1}{n+1}\sum_{i=1}^{n} X_i$.

    1. Are $\hat{\mu}_{1,n}$ and $\hat{\mu}_{2,n}$ unbiased?
    2. Are $\hat{\mu}_{1,n}$ and $\hat{\mu}_{2,n}$ consistent?

5. Consider a sample of random variables: $X_1, X_2, \ldots, X_n$, where $n > 10$, $E[X_i] = \mu$, $\text{Var}[X_i] = \sigma^2 > 0$ and the estimator $\hat{\mu}_n = \frac{1}{n-10}\sum_{i=11}^{n} X_i$. Then calculate:

    1. The bias of $\hat{\mu}_n$.
    2. The variance of $\hat{\mu}_n$.
    3. The MSE of of $\hat{\mu}_n$.