

Probability and Statistics 1. Exercises 6

Convention: $[n]$ stands for $\{1, \dots, n\}$. * indicates a bonus question for students interested to explore the topic in more depth.

1. For a continuous random variable X , express the following in terms of the distribution function F_X , and the density function f_X (separately):
 - $\mathbb{P}[X \in (0, 1]]$,
 - $\mathbb{P}[X < 0]$,
 - $\mathbb{P}[X > 0]$,
 - $\mathbb{P}[X \in [0, 1]]$.
2. We throw darts at a circular board that has radius 1 (any units). Assume that every point in the target has an equal probability of being hit. More precisely, each subset has a probability proportional to its area. Let X denote the distance from the center.
 - Find the distribution function F_X .
 - Find the density function f_X .
 - Compute $E[X]$, $\text{Var}[X]$ and σ_X .
3. The average lifespan of a hard drive is 4 years. Assume that this time is described by a random variable with an exponential distribution.
 - What is the probability that the drive will fail in the first three years?
 - What is the probability that it will last at least ten years?
 - After what time will 10% of disks break?
4. Prove LOTUS for continuous random variable. (Hint: Assume $g(X)$ is a differentiable function.)
5. Let the current flowing through a 1-ohm resistor I be a random variable uniformly distributed on the interval (a, b) , for $a, b > 0$. If the power dissipated by this resistor is $X = I^2$. What is the average power dissipated by the resistor? (Hint: Use LOTUS.)

6. Consider the cumulative distribution given by

$$P(x) = \frac{1}{\pi} \frac{b}{b^2 + (x - m)^2}.$$

Show that this distribution does not have mean. [This is called Cauchy distribution; it can be described as the distribution of horizontal distances at which a line segment tilted at a random angle cuts the x-axis.]

7. * Suppose we have a floor made of parallel strips of wood, each the same width, and we drop a needle onto the floor. What is the probability that the needle will lie across a line between two strips?