Probability and Statistics 1. Exercises 4

Convention: $[n]$ stands for $\{1, \ldots, n\}$. * indicates a bonus question for students interested to explore the topic in more depth.

1. A biased die has the probability distribution described in the table. What is the expected value of X where X is the number thrown? Find the variance of X.

Number	1	2	3	4	5	6
Probability	0.3	0.2	0.25	0.15	0.05	0.05

2. Consider a biased coin such that the probability of getting a heads is p. Let the random variable be the number of coin tosses required until the first head. Find the expectation and the variance of this random variable.

3. Let X be a discrete random variable with the Poisson distribution with parameter λ. Show that the expectation X is λ. What is the variance of X?

4. Consider the infinite die, such its sides are numbered by $\mathbb{N}$ and define the probability mass function as $P(X = m) = C/m^3$, for some constant C. Let X be the random variable that you roll the number m. Show that $E[X]$ is finite. Does the variance of X exist?

5. Let X be a random variable with values in $\mathbb{N}$. Show that $E[X] = \sum_{n=0}^{\infty} P(X > n)$.

6. * There are n different types of pokemons in a game. Each time you play, you get one random pokemon. Every pokemon is equally likely to appear. How many times do you need to play on average to collect all the pokemons? How fast does the expecation grow?