Probability and Statistics 1. Exercises 10

1. Find the MLE (most likelihood estimator) of θ if $x_1, \ldots, x_n$ are iid samples from

    1. $\text{Exp}(\theta)$
    2. $\text{Unif}(0, \theta)$

2. What are the MoM (Method of moments) estimator of θ for the previous problem?

3. Suppose $x_1, \ldots, x_n$ are iid samples from $\text{Poi}(\theta)$ where θ is unknown. The MLE and MoM estimates agree at the sample mean: $\hat{\theta} = \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$. Create an interval centered at $\hat{\theta}$ which contains θ with probability 95%.

4. You want to determine whether or not more than 3/4 of Americans would vote for Donald Trump for President in 2024 (if he still will be alive). In a random poll sampling $n = 137$ Americans, we collected responses $x_1, \ldots, x_n$ (each is 1 or 0, if they would vote for him or not). We observe 131 "yes" responses: $\sum_{i=1}^{n} x_i = 131$. Perform a hypothesis test and state your conclusion.