Homework 2: Probability and Statistics 1

Due: 21 May 2023

1. A coin is biased such that the probability of getting a head is $\frac{1}{10}$. This coin is flipped 200 times. Use Markov's inequality to give an upper bound on the probability that the coin lands heads at least 120 times. Improve this bound using Chebyshev's inequality.

2. Based on a random sample of size n, find the maximum likelihood estimator of the parameters

    (a) θ for $f(x \mid \theta) = \theta e^{-\theta x}$, $x > 0$.

    (b) p for $f(x \mid p) = p(1-p)^x$, $x = 0, 1, 2, \ldots$.

3. Based on some random sample of size m, use the method of moments to estimate the parameters

    (a) θ for the density $f(x \mid \theta) = \frac{1}{\theta}$, $0 < x < \theta$.

    (b) n, p for $f(x \mid \theta) = \binom{n}{x} p^x (1-p)^{n-x}$, $n \in \mathbb{N}$, $0 \le x \le n$.

4. Given $\bar{X} = 40$ for a random sample of size 50 from the density $f(x \mid \mu) = \frac{\exp(-[x-\mu]^2/32)}{4\sqrt{2\pi}}$, find a 90% and 95% confidence interval for μ, where $\exp(x) = e^x$.

5. Let H_0 be the hypothesis that $p = 1/2$ and H_1 be the hypothesis that $p = 2/3$, where p is the probability of success in an experiment. The experiment is performed twice. If it is agreed to accept H_0 if and only if, two successes are obtained, what are the values of α (size of type-I error, that is, P[accept $H_1 \mid H_0$]) and β (size of type-II error, that is, P[accept $H_0 \mid H_1$])?