Notes on the exam from Linear algebra II

The exam consists of testing knowledge of three definitions, one theorem with its proof and a survey question of one topic. Each definition is followed by a simple problem to apply the definition. The list is not exhaustive nor obligatory — knowledge of concepts and facts not mentioned in this list may be asked as well.
Besides others this includes all essential concepts from winter term like: matrices, their properties and related operations; groups and fields; vector spaces; bases; linear maps and their matrices.

Define a permutation.
Define the sign of a permutation.
Define the determinant.
Define the adjoint matrix.
Define the Laplace matrix.
Define a polynomial over a field.
Define the root of a polynomial and its multiplicity.
Define an algebraically closed field.
Define the Vandermonde matrix.
Define the eigenvalue and eigenvector of a linear map.
Define the eigenvalue and eigenvector of a matrix.
Define the characteristic polynomial.
Define the algebraic multiplicity of an eigenvalue.
Define the geometric multiplicity of an eigenvalue.
Define similar matrices.
Define a diagonalizable matrix.
Define a Jordan block.
Define a Jordan normal form of a matrix.
Define a generalized eigenvector.
Define a Hermitian matrix.
Define a unitary matrix.
Define an inner product over complex vector spaces.
Define the norm associated with an inner product.
Define orthogonal vectors.
Define an orthonormal basis.
Define Fourier coefficients.
Define the orthogonal projection.
Define an isometry.
Define the orthogonal complement.
Define the Gram matrix.
Define a positive definite matrix.
Define the Cholesky factorization.
Define a bilinear form.
Define a quadratic form.
Define the matrix of a bilinear for w.r.t. a basis.
Define the analytic expression of a form.
Define the signature a form.

State and prove a theorem about the linearity of the determinant.
State and prove a theorem about the determinant of the product of two matrices.
State and prove a theorem about the Laplace expansion of the determinant.
State and prove a the Cramer's rule (solving systems with determinants).
State and prove a theorem about the adjoint matrix.
State and prove a theorem about the number of spanning trees of a graph.
State and prove the Fermat's little theorem.
State and prove a theorem about Vandermonde matrix.
State and prove correctness of the Lagrange interpolation.
State and prove a theorem about the subspace of eigenvectors.
State and prove a theorem about linear independence of eigenvectors.
State and prove a theorem about the roots of the characteristic polynomial.
State and prove the Cayley-Hamilton theorem.
State and prove a necessary and sufficient condition when a matrix is diagonalizable.
State and prove a theorem about diagonalization of special complex matrices.
State and prove the Cauchy-Schwarz inequality.
State and prove the triangle inequality.
State and prove a theorem about Fourier coefficients.
State and prove correctness of Gram-Schmidt orthonormalization (including the lemma, if you need it).
State and prove a theorem about an isometry and norm.
State and prove a theorem about an isometry and the properties of its matrix.
State and prove a theorem about the orthogonal complement.
State and prove a theorem about the inner product of two vectors and Gram matrix.
State and prove a theorem about three equivalent conditions on positive definite matrices.
State and prove a theorem about a recurrent condition on positive definite matrices.
State and prove a theorem about positive definite matrices and determinants.
State and prove correctness of Cholesky factorization.
State and prove a theorem about the diagonalization of matrices of forms.
State and prove the Sylvester's law of inertia – about diagonalization of quadratic forms.
State and prove a theorem about the number of lines spanning the same angle.

Write a summary about the calculation of determinants.
Write a summary about the determinnats and their geometric meaning.
Write a summary about the number of spanning trees of a graph.
Write a summary about polynomials.
Write a summary about eigenvalues and eigenvectors.
Write a summary about characteristic polynomial and its coefficients.
Write a summary about similar matrices and diagonalization.
Write a summary about special complex matrices.
Write a summary about inner spaces and the associated norms.
Write a summary about orthogonality and orthogonal projection.
Write a summary about orthonormal bases.
Write a summary about orthogonal complement.
Write a summary about positive definite matrices.
Write a summary about bilinear and quadratic forms and their matrices.

(For survey questions please provide definitions, theorem statements, examples and relationships. Proofs are not required.)

Last update May 13, 2022.