Notes to the Linear algebra exam

The exam consists of testing knowledge of three definitions, one theorem with its proof and a survey question of one topic. Each definition is followed by a simple problem to apply the definition. The list is not exhaustive nor obligatory — knowledge of concepts and facts not mentioned in this list may occasionally be asked as well.

Define augmented matrix.
Define elementary row operations.
Define row echelon form of a matrix.
Write a pseudocode for the Gaussian elimination.
Define free and dependent variables.
Define the rank of a matrix.
Define the identity matrix.
Define the transpose matrix.
Define a symmetric matrix.
Define the matrix product.
Define the inverse matrix.
Define a regular matrix.
Define a binary operation.
Define the commutative and associative binary operations.
Define the neutral element.
Define the inverse element.
Define a group.
Define a permutation.
Define a transposition.
Define an inversion of a permutation.
Define the sign of a permutation.
Define a field.
Define the characteristics of a field.
Define a vector space.
Define a subspace of a vector space.
Define a linear combination.
Define a linear hull (a subspace generated by a set).
Define the row and the column space of a matrix.
Define the kernel of a matrix.
Define linearly independent vectors.
Define a basis of a vector space.
Define the dimension of a vector space.
Define the vector of coordinates.
Define a linear map.
Define the matrix of a linear map.
Define the change of basis matrix.

Define an isomorphism of vector spaces.
Define an affine space and its dimension.

State and prove a relationship between elementary row operations and systems of equations.
State and prove a theorem about the uniqueness of free and dependent variables.
State and prove the Frobenius theorem.
State and prove a theorem about the relationship between solutions of $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Ax} = \mathbf{0}$.
State and prove a theorem describing all solutions of $\mathbf{Ax} = \mathbf{b}$.
State and prove a theorem on equivalent definitions of regular matrices.
State and prove a theorem on the sign of the composed permutation.
State and prove a theorem characterizing when $\mathbb{Z}_n$ is a field.
State and prove the Fermat's little theorem.
State and prove a theorem about an intersection of vector spaces.
State and prove a theorem on equivalent definitions of linear hull.
State and prove the Steinitz's exchange theorem (including the lemma if you need it).
State and prove a theorem about the uniqueness of a linear map.
State and prove a theorem about a characterization of an isomorphism between vector spaces.
State and prove a theorem about vector spaces related to a matrix $\mathbf{A}$.

Write a summary about elementary row operations and Gaussian elimination.
Write a summary about solving homogeneous and non-homogeneous systems of linear equations.
Write a summary about matrix operations.
Write a summary about regular and singular matrices.
Write a summary about binary operations and their properties.
Write a summary about (general) groups.
Write a summary about permutation groups.
Write a summary about fields.
Write a summary about vector spaces and their subspaces.
Write a summary about vector spaces related to a matrix $\mathbf{A}$.
Write a summary about linear dependence.
Write a summary about bases of vector spaces.
Write a summary about linear maps and their matrices.

(For survey questions please provide definitions, theorem statements, examples and relationships. Proofs are not required.)