

Notes to the Linear algebra exam

The exam consists of testing knowledge of three definitions, one theorem with its proof and a survey question of one topic. Each definition is followed by a simple problem to apply the definition. The list is not exhaustive nor obligatory — knowledge of concepts and facts not mentioned in this list may occasionally be asked as well.

Define augmented matrix.

Define elementary row operations.

Define row echelon form of a matrix.

Write a pseudocode for the Gaussian elimination.

Define free and dependent variables.

Define the rank of a matrix.

Define the identity matrix.

Define the transpose matrix.

Define a symmetric matrix.

Define the matrix product.

Define the inverse matrix.

Define a regular matrix.

Define a binary operation.

Define the commutative and associative binary operations.

Define the neutral element.

Define the inverse element.

Define a group.

Define a permutation.

Define a transposition.

Define an inversion of a permutation.

Define a field.

Define the characteristics of a field.

Define a vector space.

Define a subspace of a vector space.

Define a linear combination.

Define a linear hull (a subspace generated by a set).

Define the row and the column space of a matrix.

Define the kernel of a matrix.

Define linearly independent vectors.

Define a basis of a vector space.

Define the dimension of a vector space.

Define the vector of coordinates.

Define a linear map.

Define the matrix of a linear map.

Define the change of basis matrix.

Define an isomorphism of vector spaces.

Define an affine space and its dimension.

State and prove a relationship between elementary row operations and systems of equations.

State and prove a theorem about the uniqueness of free and dependent variables.

State and prove the Frobenius theorem.

State and prove a theorem about the relationship between solutions of $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Ax} = \mathbf{0}$.

State and prove a theorem describing all solutions of $\mathbf{Ax} = \mathbf{b}$.

State and prove a theorem on equivalent definitions of regular matrices.

State and prove a theorem characterizing when \mathbb{Z}_n is a field.

State and prove the Fermat's little theorem.

State and prove a theorem about an intersection of vector spaces.

State and prove a theorem on equivalent definitions of linear hull.

State and prove the Steinitz's exchange theorem (including the lemma if you need it).

State and prove a theorem about the uniqueness of a linear map.

State and prove a theorem about a characterization of an isomorphism between vector spaces.

State and prove a theorem about vector spaces related to a matrix \mathbf{A} .

Write a summary about elementary row operations and Gaussian elimination.

Write a summary about solving homogeneous and non-homogeneous systems of linear equations.

Write a summary about matrix operations.

Write a summary about regular and singular matrices.

Write a summary about binary operations and their properties.

Write a summary about (general) groups.

Write a summary about permutation groups.

Write a summary about fields.

Write a summary about vector spaces and their subspaces.

Write a summary about vector spaces related to a matrix \mathbf{A} .

Write a summary about linear dependence.

Write a summary about bases of vector spaces.

Write a summary about linear maps and their matrices.

(For survey questions please provide definitions, theorem statements, examples and relationships. Proofs are not required.)