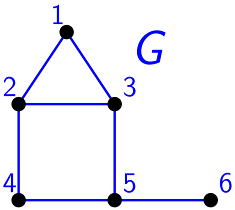
Počet koster grafu — aplikace determinantu



Laplaceova matice

$$L_G = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\left| L_G^{1,1} \right| = \begin{vmatrix} 3 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix} = 11$$

Vlastnosti determinantu matice $L^{i,j}$

$$
\left|L_G^{1,1}\right| = \begin{vmatrix} 3 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix} \begin{matrix} \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \\ \mathrm{VI} \end{matrix} =
$$

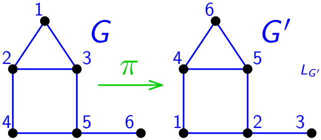
$$
L_G = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}
$$

$$
\begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix} \begin{matrix} \mathrm{II} + \mathrm{III} + \mathrm{IV} + \mathrm{V} + \mathrm{VI} \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \\ \mathrm{VI} \end{matrix} =
$$

$$
(-1) \cdot \begin{vmatrix} -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix} \begin{matrix} \mathrm{I} \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \\ \mathrm{VI} \end{matrix} = - \left|L_G^{2,1}\right| = + \left|L_G^{2,2}\right| = \cdots
$$

Laplaceovy matice izomorfních grafů

$$L_G = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$



$$L_{G'} = \begin{pmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{pmatrix}$$

$$\left|L_G^{1,1}\right| = \left|L_{G'}^{6,6}\right| = \left|L_{G'}^{1,1}\right|$$

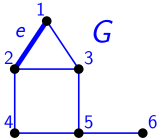
$$\begin{vmatrix} 3 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 & -1 \\ 0 & -1 & 0 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & -1 & -1 \\ -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{vmatrix}$$

Matice $L_G^{1,1}$, $L_{G'}^{6,6}$ se liší jen zpermutováním řádků i sloupců podle π.

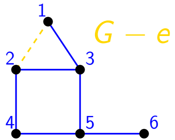
K důkazu věty

$$\kappa(G) = \kappa(G-e) + \kappa(G \circ e) = \left|L^{1,1}_{G-e}\right| + \left|L^{1,1}_{G\circ e}\right| = ??? = \left|L^{1,1}_{G}\right|$$

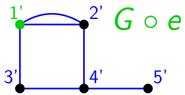
$$L_G = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$



$$L_{G-e} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$



$$L_{G\circ e} = \begin{pmatrix} 3 & -2 & -1 & 0 & 0 \\ -2 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$



K důkazu věty

$$\kappa(G) = \kappa(G-e) + \kappa(G \circ e) = \left|L^{1,1}_{G-e}\right| + \left|L^{1,1}_{G\circ e}\right| = ??? = \left|L^{1,1}_{G}\right|$$

$$L_G = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \qquad L^{1,1}_G$$

$$L_{G-e} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \qquad L^{1,1}_{G-e}$$

$$L_{G\circ e} = \begin{pmatrix} 3 & -2 & -1 & 0 & 0 \\ -2 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \qquad L^{1,1}_{G\circ e}$$

K důkazu věty

$$\kappa(G) = \kappa(G-e) + \kappa(G \circ e) = \left|L^{1,1}_{G-e}\right| + \left|L^{1,1}_{G\circ e}\right| \quad = \quad \left|L^{1,1}_{G}\right|$$

$$L_G = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 + 1 & -1 & -1 & 0 & 0 \\ -1 & -1 + 0 & 3 & 0 & -1 & 0 \\ 0 & -1 + 0 & 0 & 2 & -1 & 0 \\ 0 & 0 + 0 & -1 & -1 & 3 & -1 \\ 0 & 0 + 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad L^{1,1}_G$$

$$L_{G-e} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad L^{1,1}_{G-e}$$

$$L_{G\circ e} = \begin{pmatrix} 3 & -2 & -1 & 0 & 0 \\ -2 & 3 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad L^{1,1}_{G\circ e}$$