Tomáš Masařík
masarik@kam.mff.cuni.cz

Presented paper by Fedor V. Fomin, Alexander Golovnev, Alexander S. Kulikov, Ivan Mihajlin

Tight Bounds for Subgraph Isomorphism and Graph Homomorphism

(http://arxiv.org/abs/1507.03738)

Known useful results

Exponential Time Hypothesis There is a positive real s such that 3-SAT with n variables and m clauses cannot be solved in time $2^{sn}(n+m)^{\mathrm{O}(1)}$.

Definition *[Graph homomorphism]*

Let G be an n vertex graph and H be an $h(n)$ vertex graph.

Then vertex mapping is graph homomorphism $G \hookrightarrow H$ iff every edge of G is mapped to an edge in G.

Lemma *[ETH for 3-*Coloring *(Lokhstanov et al. 2013)] Unless ETH fails, there exist a constant $q > 0$ such that there is no algorithm solving 3-*coloring *on a graph with n vertices of maximum degree four in time $\mathrm{O}(2^{qn})$.*

Lemma *[lemma 5 (Fomin et al. 2015)] There is a polynomial-time algorithm that from an instance (G, H) of* List Graph Homomorphism *construct an instance (G', H') of* Graph Homomorphism *such that $|V(G')| \leq |V(G)| + 25|V(H)|^2$ and $|V(H')| \leq 25|V(H)|^2$.*

New results

Theorem *[Tightness of graph homomorphism] Let G be an n vertex graph and H be an $h(n)$ vertex graph.*

Unless ETH fails, for any constant $D \geq 1$ there exists a constant $c(D) > 0$ such that for any function h satisfying $3 \leq h(n) \leq n^D$ there is no $\mathrm{O}(h^{cn})$-time algorithm deciding $G \hookrightarrow H$.