Solution to Problem 2.4: Assume $B = \{b_1 > b_2 > \cdots > b_n > 3\}$, otherwise traslate the set, and take $c = 2b_1$. Taking operations modulo c we can verify that in the interval $[\frac{3c}{9}, \frac{4c}{9})$ there are no four elements b_1, b_2, b_3, b_4 such that $b_1 + 2b_2 = 2b_3 + 2b_4 \mod c$. To see this we notice that the left hand side, i.e. $b_1 + 2b_2$, takes values in the interval $[0, \frac{c}{9})$, while the right hand side, i.e. $2b_3 + 2b_4$, takes values in the interval $[\frac{c}{9}, \frac{7c}{9})$. Because this two intervals are disjoint there are not such four elements satisfying the equality. Let $x \in [0, c)$ picked uniformly at random, i.e. for any $0 \leq a < b \leq c$ we have that $\Pr[x \in [a, b]] = \frac{b-a}{c}$. Let $b \in B$, then

$$\Pr\left[xb \in \left[\frac{3c}{9}, \frac{4c}{9}\right]\right] \geq \Pr\left[x \in \bigcup_{k=1}^{k_b} \left[\frac{3c}{9b} + \frac{kc}{b}, \frac{4c}{9b} + \frac{kc}{b}\right]\right]$$

To see this let $1 \leq k \leq k_b$ and $x \in [\frac{3c}{9b} + \frac{kc}{b}, \frac{4c}{9b} + \frac{kc}{b}]$, then $xb \in [\frac{3c}{9} + kc, \frac{4c}{9} + kc] = [\frac{3c}{9}, \frac{4c}{9}]$ where the last equality follows from taking modulo c.

Next we verify that the intervals $[\frac{3c}{9b} + \frac{kc}{b}, \frac{4c}{9b} + \frac{kc}{b}]$ are in $[0, c)$. On the one hand $\frac{3c}{9b} + \frac{kc}{b} < c$ iff $k < b - 1/3$ and $\frac{4c}{9b} + \frac{kc}{b} < c$ iff $k < b - 4/9$. It is enough to take $k_b = b - 2$. If this is the case, then $\Pr[x \in [\frac{3c}{9b} + \frac{kc}{b}, \frac{4c}{9b} + \frac{kc}{b}]] = \frac{1}{9b}$.

Finally we verify that the intervals are disjoint. It is enough to see that for consecutive values of k they are disjoint, that is $\frac{4c}{9b} + \frac{kc}{b} < \frac{3c}{9b} + \frac{(k+1)c}{b}$. This happens if and only if $1/9 \leq 1$, which is true.

Putting it all together, we obtain that

$$\Pr[xb \in [\frac{3c}{9}, \frac{4c}{9}]] \geq \frac{(b-2)}{9b} \geq 1/9,$$

where we have used that $b \geq 3$. Now, let X_i be the random variable for the event $xb_i \in [\frac{3c}{9}, \frac{4c}{9})$, and set $X = \sum_{i=1}^{n} X_i$. By the above computations, $E[X] \geq \frac{n}{9}$ and consequently there exists an x such that at least $n/9$ elements land in the interval $[\frac{3c}{9}, \frac{4c}{9})$.