

Probabilistic techniques - tutorials

Classwork 6 – Markov chains

Definition 1. Let $r_{i,j}^t = \Pr[X_t = j \text{ and } X_s \neq j \text{ for all } 0 < s < t | X_0 = i]$. A state i is recurrent if $\sum_t r_{i,i}^t = 1$ and it is transient if $\sum_t r_{i,i}^t < 1$. A communicating class C is closed if for all $i \in C$ it holds that if i communicates with j then j is in C as well.

1. Classification of states.
 - (a) A state is recurrent if and only if $\sum_n P_{i,i}^n = \infty$.
 - (b) A state is recurrent if and only if $\Pr[X_n = i \text{ for infinitely many } n | X_0 = 1] = 1$.
 - (c) A state i is transient if and only if $\sum_n P_{i,i}^n < \infty$.
 - (d) A state i is transient if and only if $\Pr[X_n = i \text{ for infinitely many } n | X_0 = i] = 0$.
2. Let i be a recurrent state and assume that i communicates with j , then j communicates with i . In particular recurrent communicating classes are closed.
3. Every finite closed communicating class is recurrent.