

Probabilistic techniques - tutorials

Classwork 3 – The method of alternation

1. For any integer n , show that $R(k, k) > n - \binom{n}{k} 2^{1-\binom{k}{2}}$. Can you write the expression on the right hand in terms of k ?
2. A dominating set of an undirected graph $G = (V, E)$ is a set $U \subseteq V$ such that every vertex $v \in V \setminus U$ has at least one neighbor in U . Let $G = (V, E)$ be a graph on n vertices, with minimum degree $\delta > 1$. Then G has a dominating set of at most $n \frac{1+\ln(\delta+1)}{\delta+1}$ vertices.
3. Let $G = (V, E)$ be a graph on n vertices with minimum degree $\delta > 10$. Prove that there is a partition of V into two disjoint sets A and B such that $|A| \leq \mathcal{O}(n \frac{\ln(\delta)}{\delta})$ and each vertex in B has at least one neighbor in A and at least one neighbor in B .
4. Let $m(n) > m$, given any n -uniform hypergraph $H = (V, E)$ with m edges, there exists a two-coloring of V such that no edge is monochromatic. Show that $m(n) = \Omega(2^n (n/\ln(n))^{1/2})$.