Probabilistic techniques - tutorials

Classwork 2 – Linearity of expectation

1. Show that there is a two coloring of the edges of K_n with at most $\binom{n}{a}2^{1-\binom{a}{2}}$ monochromatic K_a.
2. Show that there is a two coloring of edges of $K_{n,m}$ with at most $\binom{n}{a}\binom{m}{b}2^{1-ab}$ monochromatic $K_{a,b}$.
3. Let M be an $n \times n$ matrix with entries uniformy independent chosen from $\{-1, 1\}$. Determine $\mathbb{E}[\det(M)]$.
4. Show that every set of integers size n has a subset A of size at least $n/3$ such that there is no triple $a_1, a_2, a_3 \in A$ that satisfies $a_1 + a_2 = a_3$.
5. Let $n \geq 2$, $H = (V, E)$ an n-uniform hypergraph with $|E| = 4^{n-1}$ edges. Show that there is a coloring of V by four colors such that no edge is monochromatic.
6. Show that every set of real numbers of size n has a subset A of size at least $n/3$ such that there is no triple $a_1, a_2, a_3 \in A$ that satisfies $a_1 + a_2 = a_3$.
7. Let F be a family of subsets of $[n] = \{1, \ldots, n\}$ and suppose that there are no $A, B \in F$ such that $A \subseteq B$. Let $\sigma \in S_n$ be a random permutation of $[n]$ and consider the random variable $X = |\{i \colon \{\sigma(1), \ldots, \sigma(i)\} \in F\}|$. Show that $|F| \leq \binom{n}{\lfloor n/2 \rfloor}$ using $\mathbb{E}[X]$.

Information about tutorials `https://kam.mff.cuni.cz/~dbulavka/teaching/ws2223/pt.html`