Probabilistic techniques - tutorials

Classwork 1 – Basics

1. Prove that there exist constants $c_1, c_2 > 0$ such that for every integers n and m the following holds:
   (a) If $n > c_1 m^2$, then a random mapping $[n] \to [m]$ is surjective with probability at least 0.99.
   (b) If $n < c_2 m^2$, then a random mapping $[n] \to [m]$ is surjective with probability at most 0.01.
2. If $\binom{n}{k}(1-2^{-k})^{n-k} < 1$, then there exist a tournament $T = (V, E)$ with $|V| = n$ that satisfies the following. For every subset $U \subset V$ of size k there exists $v \in V \setminus U$ such that $(v, u) \in E$ for all $u \in U$.
3. Find an example of three non-empty events A, B and C in classical probability space that are not independent, but it holds that $\Pr[A \cap B \cap C] = \Pr[A]\Pr[B]\Pr[C]$.
4. Let $\{(A_i, B_i) \colon i = 1, \ldots, h\}$ such that $|A_i| = k$, $|B_i| = l$, $A_i \cap B_i = \emptyset$ and $A_i \cap B_j \neq \emptyset$ for $i \neq j$. Show that $h \leq \binom{k+l}{k}$.

Information about tutorials `https://kam.mff.cuni.cz/~dbulavka/teaching/ws2223/pt.html`