

Mathematics++

Practicals 3 – Minkowski Sum and Volume in High Dimension

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Definition: Let $A, B \subseteq \mathbb{R}^n$. The *Minkowski sum* is

$$A \oplus B := \{a + b : a \in A, b \in B\}.$$

1. Determine Minkowski sum (in \mathbb{R}^2) of sets $[0, 1] \times \{0\}$ and $\{(x, x) : x \in [0, 1]\}$.
2. Determine whether Minkowski sum of balls is a ball.
3. Determine whether Minkowski sum of ellipsoids (where ellipsoid is a linear transformation of a ball) is an ellipsoid.
4. Show that Minkowski sum of convex sets is convex. Does the implication in the other direction hold?
5. Show that Minkowski sum of compact sets is compact.
6. Show that for every $V \subseteq \mathbb{R}^n$ the following holds: [*]

$$\text{conv}(V) \subseteq \bigcup_{v \in V} B\left(\frac{1}{2}v, \frac{1}{2}\|v\|\right)$$

7. Show that convex hull of k points contained in a ball $B \subseteq \mathbb{R}^n$ has volume at most $\frac{k}{2^n} \lambda(B)$.
Hint: Use previous exercise.